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ANALYTIC MODEL OF BEB ALGORITHM WITH MULTIPLE PRIORITIES IN MOBILE INFORMATION SYSTEMS

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Abstract. In this paper we propose analytic model for computing the delay of the slotted ALOHA protocol with Binary Exponential Backoff (BEB) with multiple priorities as a collision resolution algorithm in mobile information systems. If a packet which tries to reserve a channel collides times, it chooses one of the next 2^n frames with equal probabilities and attempts the reservation again. We derive the expected access delay until an arbitrary packet reserves a channel in any cell. Then the expected transmission delays for packets of calls with multiple priorities are calculated analytically. Proposed analytic model is checked against simulation.

Keywords: Analytic model, slotted ALOHA protocol, BEB, access delay, transmission delay, mobile information systems

1 INTRODUCTION

Slotted ALOHA(S-ALOHA) protocol has been widely adopted in local wireless/mobile communication systems as a random multiple access protocol. In these systems, each frame is divided into small slots and each Mobile Terminal (MT) contends for the slot to transmit its packets at the beginning of each frame. If two or more MTs contend for the same slot, then a collision occurs and none of them can transmit their packets of calls. The colliding packets are queued and retry after a random delay. The way to resolve the collision is called the collision resolution protocol. One of the widely used collision resolution protocols is the BEB algorithm, forms of which are included in Ethernet and Wireless LAN (WLAN) standards. Whenever a node's packet is involved in a collision, it selects one of the next 2^n frames with equal probabilities, where n is the number of collisions that the packet has ever experienced, and attempts the retransmission.

Soni and Chockalingam [5] analyzed three backoff schemes, namely, linear backoff, exponential backoff, and geometric backoff. They calculated the throughput and energy efficiency as the reward rates in a renewal process and illustrated that the truncated BEB, which is considered in this paper, performs better since the idle length should grow only until a maximum value by numerical result. More recently, Chen and Li [7] proposed the Quasi-FIFO algorithm, which is another novel collision resolution scheme. They showed, by simulation, that the proposed scheme shares the bandwidth more equally and maximizes the throughput, but no analytic model was given in [7] as well.

Delay distributions of slotted ALOHA and CSMA are derived in Yang and Yum [8] under three retransmission policies. They found the conditions for achieving finite delay mean and variance under the BEB. Their assumption, however, that the combination of new and retransmitted packet arrivals is a Poisson process is not valid because the stream of the retransmitted packets depends on the arrivals of new packets. This dependency makes the Poisson assumption invalid. Chatzimisios and Boucouvalas [9] presented an analytic model to compute the throughput of the IEEE 802.11 protocol for WLAN and examined the behavior of the Exponential Backoff (EB) algorithm used in IEEE 802.11. They assumed that the collision probability of a transmitted frame is independent of the number of retransmissions. As we will show later in this paper, however, this probability is a function of the number of competing stations and also depends upon the number of retransmissions that this station has ever experienced. Kwak et al. [10] gave new analytical results for the performance of the EB algorithm. Especially, they derived the analytical expression for the saturation throughput and expected access delay of a packet for a given number of nodes. Their EB model, however, assumes that the packet can retransmit infinitely many times.

Stability is another issue on BEB algorithm and there are many methods dealing with this. As pointed in Kwak et al., however, these works show contradictory results because some of them do not represent the real system and they adopt different definitions of stability used in the analyses. The dispute is still going on so we do not focus on this topic but on the analytic model to analyze the performance of the BEB algorithm.

In this paper we propose analytical model to find the performance measures to evaluate the system which adopts BEB algorithm. We derive the expected access delay until an arbitrary packet reserves a channel in any cell. Then the expected transmission delay for packets with multiple priorities is calculated analytically.

This paper is organized as follows. We describe the BEB algorithm under consideration in Section 2. Section 3 describes analytic model that has the steady state distribution of the number of calls at the beginning of the frame and the collision probability. In Section 4, we derive the expected access delay and the expected transmission delay of a packet according to multiple priorities. Section 5 provides some numerical results to exemplify our proposed method. Finally, we conclude the paper in Section 6.

2 MOBILE SYSTEM MODEL

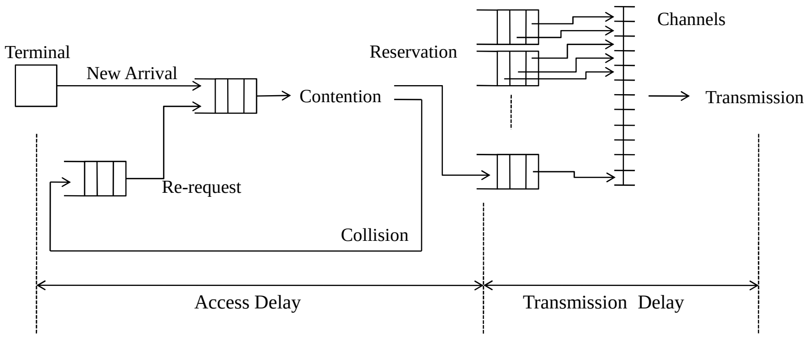

Fig. 1. Access and transmission procedure in mobile information systems

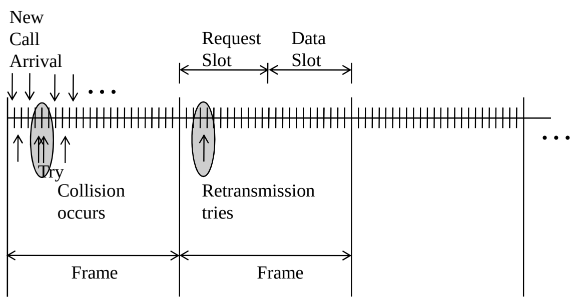

Fig. 2. Collision model

Figure 1 shows the procedure considered in this paper – the access delay and transmission delay with multiple priority calls in any cell of mobile information systems. We assume that new packets arrive from infinite number of MTs forming a Poisson process with arrival rate λ to the system. The time is divided into slots which are grouped into frames of fixed size. A frame is divided into two groups of multiple slots, request slots for reservation of channel and transmission slots for transmission of the actual information. The numbers of request slots and transmission slots in a frame are V and T, respectively.

The types of packet are divided into multiple call traffics such as image, voice, data, and so on. We assume p multiple traffic types of packet indexed $i = 1, 2, \ldots, p$. Newly arrived packet is assumed to be one of p multiple call traffic types with probability $\alpha_i(\sum_i^p \alpha_i = 1)$. It is essential that a priority is given to the lower indexed call traffic type, but an effort is made to accommodate the higher indexed call traffic type, whenever possible. When a packet of multiple call traffics (regardless of priority) arrives at the system, it waits until the beginning of the next frame and randomly accesses one of the request slots to reserve a channel for transmission. If the packet succeeds in the reservation, then a channel is allocated in any cell. If, however, two or more packets contend for the same request slot, then a collision occurs and none of the packets can reserve the request slot. Figure 2 shows a frame structure and the collision model.

The packet which fails to get a request slot retries under the BEB algorithm: Whenever a packet is involved in a collision and if it was the b^{th} ($b = 0, 1, \ldots, 15$) collision, then it selects one of the next 2^i frames with probability $1/2^i$ and attempts the reservation again, where $i = \min(b, 10)$. If a packet collides 16 times, then it fails to transmit and is dropped. Those packets reserved slots then enter the queues and transmit themselves according to the proper scheduling method. Figure 3 depicts the BEB algorithm considered here. As stated in the introduction the system is assumed to be in the stable condition.

3 ANALYTIC MODEL

We obtain the Steady State Distribution (SSD) of the number of packets at the beginning of the frame. Let A_n be the number of new packets arrived during the n^{th} frame and be the total number of packets waiting in the system at the beginning of the n^{th} frame. Also, denote by the number of packets which successfully reserve a request slot at the n^{th} frame. Then it can be shown that

$$N_{n+1} = \begin{cases} N_n - J_n + A_n, & N_n \geq 1 \\ A_n, & N_n = 0 \end{cases} \tag{1}$$

and $\{N_n, n \geq 1\}$ is a Markov chain process. Let us denote a_j, $j = 0, 1, 2, \ldots$ by the SSD of A_n, where $a_j = \Pr(A_n = j) = e^{-\lambda d}(\lambda d)^j/j!$, $j = 0, 1, 2, \ldots$, and d is the length of a frame. We obtain the one step transition probabilities $p_{ij} = \Pr(N_{n+1} = j|N_n = i)$, $i, j = 0, 1, 2, \ldots$ of the Markov chain process as follows:

$$\begin{aligned} P_{ij} &= \Pr(N_{n+1} = j|N_n = i) \\ &= \sum_{k=\max(0,i-j)}^{\min(i,V)} \Pr(N_{n+1} = j, J_n = k|N_n = i) \\ &= \sum_{k=\max(0,i-j)}^{\min(i,V)} \Pr(N_{n+1} = j|J_n = k, N_n = i)\Pr(J_n = k|N_n = i) \qquad (2) \\ &= \sum_{k=\max(0,i-j)}^{\min(i,V)} a_{j-i+k}\Pr(J_n = k|N_n = i), \end{aligned}$$

for, $i \geq 1$ and $p_{oj} = a_j$. In order to compute the probability $\Pr(J_n = k|N_n = i)$, the probability that k calls are successful in contending the request slots out of i calls, let us introduce a random variable Y_n that is the number of calls which actually participate in the contention at the n th frame. Then we have

$$\begin{aligned}\Pr(J_n = k|N_n = i) &= \sum_{y=0}^{i} \Pr(J_n = k|Y_n = y, N_n = i)\Pr(Y_n = y|N_n = i) \\ &= \sum_{y=0}^{i} \Pr(J_n = k|Y_n = y)\Pr(Y_n = y|N_n = i). \qquad (3)\end{aligned}$$

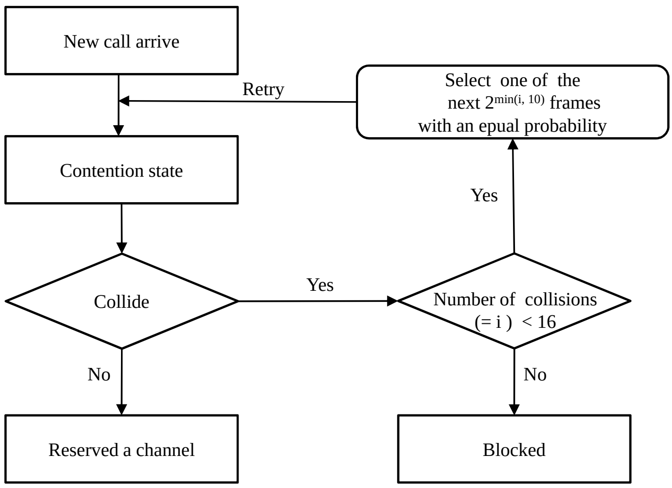

Fig. 3. Flowchart of BEB algorithm

Let us denote $J(y,k) = \lim_{x\to\infty} \Pr(J_n = k|Y_n = y)$ and $Y(y,k) = \lim_{x=\infty} \Pr(Y_n = y|N_n = i)$. Then $J(y,k)$ is the probability that k calls succeed in the contention among y competing calls and $Y(i,y)$ is the probability that y calls participate in the contention among i calls in the steady state. Then $J(y,k)$ is derived in Szpankowski [13], for $0 \leq k \leq \min(v,y)$, as follows:

$$J(y,k) = \frac{(-1)^k v! y!}{v^y k!} \sum_{m=k}^{\min(v,y)} (-1)^m \frac{(v-m)^{y-m}}{(m-k)!(v-m)!(y-m)!}. \qquad (4)$$

Also, the probability $Y(i,y)$ is given by:

$$Y(i,y) = \binom{i}{y} r^y (1-r)^{i-y}, \quad y = 0, 1, \ldots i, \qquad (5)$$

where r is the probability that an arbitrary call message participates in the contention. Since each call waiting in the system has experienced different number of collisions (let us call this number the backoff state of the call message) we derive the probability γ by conditioning the backoff state.

Let us define an indicator random variable I_n which is 1 if a random call participates in the contention at the n th frame, and 0, otherwise. In addition, let B_n be the backoff state of a random call at the n^{th} frame. Then

$$\Pr(I_n = 1|B_n = b) = \begin{cases} (1/2)^b, & 0 \le b \le 10 \\ (1/2)^10, & 11 \le b \le 15 \end{cases} \tag{6}$$

and

$$\Pr(B_n = b) = \frac{(1-\gamma_c)\gamma_c^b}{\sum_{l=0}^{15}(1-\gamma_c)\gamma_c^l} = \frac{(1-\gamma_c)\gamma_c^b}{(1-\gamma_c)\gamma_c^{16}}, \quad b = 0, 1, \ldots, 15 \tag{7}$$

where is the probability that an arbitrarily chosen (tagged) message experiences a collision when it contends for a request slot. We derive this unknown probability in the next subsection. Then we have

$$\begin{aligned} \gamma &= \Pr(I_n = 1) \\ &= \sum_{b=0}^{15} \Pr(I_n = 1|B_n = b)Pr(B_n = b) \\ &= \left(\frac{1-(\gamma_c/2)^{11}}{1-(\gamma_c/2)}\right)\frac{1-\gamma_c}{1-\gamma_c^{16}} + (\gamma_c/2)^{10}\frac{\gamma_c(1-\gamma_c^5)}{1-\gamma_c^{16}}. \end{aligned} \tag{8}$$

Now we can calculate the one-step transition probability p_{ij}=$\Pr(N_{n+1} = j|N_n = i)$ in Equation (2) by plugging the Equation (4) and (5) combined with (8) as

$$p_{ij} = \sum_{k=\max(0,i-j}^{\min(i,V)} a_{j-i+k} \sum_{y=0}^{i} J(y,k)Y(i,y). \tag{9}$$

The Steady State Probability Distribution (SSPD) $\pi_j = \Pr(N = j) = \lim_{n\to\infty} \Pr(N_n = j)$ of the number of packets in system at the beginning of the frame can be obtained by solving the steady state equations $\pi_i = \sum_{i=0}^{\infty} \pi_i p_{ij}$ and $\sum_{i=0}^{\infty} \pi_i = 1$.

Now, we derive the collision probability γ_c that a tagged packet experiences a collision given that it actually participates in the contention for a request slot in this sub-section. This probability has not been found in an analytic form in the previous studies and we calculate it for the first time in this paper. Let M be the number of packets in the system at the beginning of the frame in which the tagged packet is included. It is known that M is differently distributed from N because it contains the tagged packet [14]. The Probability Distribution (PD) of M is given by

$$\Pr(M=m)=\frac{m\pi_m}{E(N)},\ \text{where } E(N)=\sum_{j=0}^{\infty} j\pi_j. \tag{10}$$

When y packets including the tagged packet participate in the contention, the probability that the tagged packet collides is $\sum_{i=1}^{y-1}\left(\frac{1}{v}\right)^i\left(1-\frac{1}{v}\right)^{y-1-i}$. Therefore, we have the following:

$$\begin{aligned}
\gamma_c &= \sum_{m=2}^{\infty}\sum_{y=2}^{m}\sum_{i=1}^{y-1}\begin{pmatrix} y-1 \\ i \end{pmatrix}\left(\frac{1}{v}\right)^i\left(1-\frac{1}{v}\right)^{y-1-i} Y(m,y)\Pr(M=m) \\
&= \sum_{m=2}^{\infty}\left\{1-\frac{v}{v-1}\left(1-\frac{r}{v}\right)^{m-1}+\frac{1}{v-1}(1-r)^m\right\} m\pi_m\Big/\sum_{j=0}^{\infty} j\pi_j. \qquad (11)
\end{aligned}$$

Note that the probability that a packet is eventually blocked is γ_c^{16}. In order to obtain γ_c in Equation (11), we need π_j but in turn γ_c should be given to obtain π_j. So we perform a recursive computation, i.e., we initially set γ_c to be an arbitrary value between 0 and 1 and compute π_j, $j=0,1,2,\ldots$. Then with this π_j, we update γ_c using the Equation (11) and this updated γ_c is utilized to update π_j again. This recursive computation continues until both values converge.

4 EXPECTED DELAYS

4.1 Expected Access Delay

Now we derive the expected medium access delay of a packet which is defined as the time from the moment that a packet of multiple call traffic arrives at the system to the moment that it successfully reserves a request slot. It can be obtained by counting the number of frames from which a newly arrived packet contends for a slot for the first time until it successfully reserves a slot. Figure 4 illustrates the state transition of a packet from its arrival at the system to be allocated or blocked. In the figure, T is the state that a packet successfully reserves a request slot and B the state that it is eventually blocked; the state (b,f) denotes that a message which has experienced $B=b$ $(b=0,1,\ldots,15)$ collisions is currently at the f^{th} frame among its $2^{\min(b,10)}$ candidate frames to participate in the contention. Transition probabilities are shown on each arrow.

If a packet reserves in the first trial with no collision (i.e., $b=0$), then it experiences, on average, 3/2 frame length's delay, which is the sum of 1/2 frame length (average length from the packet's arrival to the beginning of the next frame) and 1 frame length. Suppose a packet collides exactly $b(1\le b\le 10)$ times, then it selects one of 2^b states with equal probability and thus the average number of frames it has spent in the system is $\frac{1}{2}+\sum_{i=0}^{b-1}2^i+\frac{1}{2^b}\sum_{j=0}^{2^b}j$. In the same manner, if $11\le b\le 15$, the average delay is $\frac{1}{2}+\sum_{i=0}^{b-1}2^i+(b-11)\cdot 2^{10}+\frac{1}{2^{10}}\sum_{j=0}^{2^{10}}j$. We

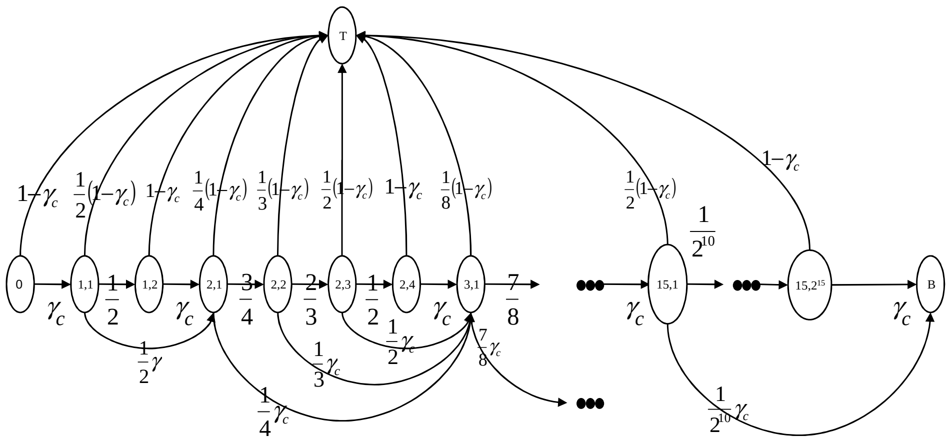

Fig. 4. State transition probability diagram

obtain the expected access delay, $E(D_{\text{Access}})$, in frames, as follows:

$$\begin{aligned} E(D_{\text{Access}}) &= \sum_{b=0}^{15} E(D_{\text{Access}}|b \text{ collisions })\Pr(b \text{ collisions}) = \frac{1}{2} + \frac{1-\gamma_c}{1-\gamma_c^{16}} \\ &\times \begin{pmatrix} 1+2.5\gamma_c+5.5\gamma_c^2+11.5\gamma_c^3+23.5\gamma_c^4+47.5_c^5+95.5\gamma_c^6 \\ +191.5\gamma_c^7+383.5\gamma_c^8+767.5\gamma_c^9+1535.5\gamma_c^{10}+2559.5\gamma_c^{11} \\ +3583.5\gamma_c^{12}+4607.5\gamma_c^{13}+5631.5\gamma_c^{14}+6655.5\gamma_c^{15} \end{pmatrix}. \end{aligned} \tag{12}$$

The PD of the number, Z, of packets which reserve request slots successfully in a frame can be obtained by conditioning Y and N as follows:

$$\Pr(Z=x) = \sum_{n=0}^{\infty}\sum_{y=0}^{n} J(y,x)Y(n,y)\pi_n. \tag{13}$$

The expected number is given by

$$\begin{aligned} E(Z) &= \sum_{n=0}^{\infty}\sum_{y=0}^{n} E(Z|Y=y,N=n)\Pr(Y=y|N=n)\pi_n \\ &= \sum_{n=0}^{\infty} nr\left(1-\frac{r}{v}\right)^{n-1}\pi_n \end{aligned} \tag{14}$$

where the second equality comes from Equation (14).

4.2 Expected Transmission Delay with Multiple Priorities

We consider the expected transmission delay of a packet, which is the time elapsed from the time that a packet succeeds in the contention until the time that it is successfully transmitted. In this paper we put a buffer of size B_i for i^{th} traffic type packet, with which one can adjust the allowable delay of the packet until its successful transmission. For example, if a longer delay for i^{th} packet is allowed with low packet dropping probability then we make B_i bigger.

Each packet uses one slot in a frame. All T transmission slots are available for packets with higher priority, so maximum T packets with higher priority can transmit simultaneously, while the packets with relatively lower priority can transmit themselves only when there are less than T slots occupied by the packets with higher priority. Therefore, no packets with relatively lower priority will be able to obtain transmission slot if there are more packets with higher priority than T at the beginning of frame.

4.2.1 Expected Transmission Delay of Packets with First Priority

Denote $K_{n,1}$ by the number of packets with first priority which succeed in the contention during the n^{th} frame and newly join at the transmission queue. Since a packet has the first priority with probability α_1, the PD of $K_{n,1}$ can be obtained from Equation (13) as follows:

$$\begin{aligned} \text{pr}(K_{n,1} = l) = k_l, l &= \sum_{x=l}^{V} \Pr(K_{n,1} = l | Z = x) \Pr(Z = x) \qquad (15) \\ &= \sum_{x=l}^{V} \binom{x}{l} \alpha_1^l (1-\alpha_1)^{x-l} \Pr(Z = x), \quad l = 0, 1, \ldots, V. \end{aligned}$$

Then $X_{n,1}$, the number of packet with first priority in the system at the beginning of the n^{th} frame, has the following relationship:

$$X_{n+1,1} = \begin{cases} K_{n,1}, & X_{n,1} \le T \\ X_{n,1} - T + K_{n,1}, & X_{n,1} > T \end{cases} \qquad (16)$$

Let $\lim_{n\to\infty} X_{n,1} = X_1, \lim_{n\to\infty} K_{n,1} = K_1$. It can be shown that $\{X_{n,1}, n \ge 1\}$ is a Markov chain process. Then we can obtain the one step transition probability matrix of this chain as follows:

$$P_1 = \begin{pmatrix} k_{1,0} & k_{1,1} & k_{1,2} & \dots & k_{1,T+B_1-1} & k_{1,T+B_1} \\ k_{1,0} & k_{1,1} & k_{1,2} & \dots & k_{1,T+B_1-1} & k_{1,T+B_1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ k_{1,0} & k_{1,1} & k_{1,2} & \dots & k_{1,T+B_1-1} & k_{1,T+B_1} \\ 0 & k_{1,0} & k_{1,1} & \dots & k_{1,T+B_1-2} & k^*_{1,T+B_1-1} \\ 0 & 0 & k_{1,0} & \dots & k_{1,T+B_1-3} & k^*_{1,T+B_1-2} \\ 0 & 0 & 0 & \dots & k_{1,T+B_1-4} & k^*_{1,T+B_1-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & k_{1,T-2} & k^*_{1,T-1} \\ 0 & 0 & 0 & \dots & k_{1,T-1} & k^*_{1,T} \end{pmatrix}, k^*_{1,i} = 1 - \sum_{m=0}^{i-1} k_{1,m}. \quad (17)$$

The SSPD, $v_1 = (v_{1,0}, v_{1,1}, \dots, v_{1,T+B_1})$, where $v_{1,j} = \Pr(X_1 = j)$, of the number of packets with first priority to be transmitted in the system at the beginning of the frame is given by solving the equations

$$\begin{aligned} v_{1,0} &= k_{1,0}\left(\textstyle\sum_{i=0}^{T} v_{1,i}\right), \\ v_{1,j} &= k_{1,j}\left(\textstyle\sum_{i=0}^{T} v_{1,i}\right) + \textstyle\sum_{m=1}^{j} v_{1,T+m}k_{1,j-m}, j \geq 1 \text{ and } \textstyle\sum_{j=0}^{T+B_1} v_{1,j} = 1. \end{aligned} \quad (18)$$

Then we can calculate the expected number of packets with first priority at the beginning of the frame, that is

$$L_1 = \sum_{j=0}^{T+B_1} j v_{1,j}. \quad (19)$$

The expected transmission delay of packets with first priority in frames is obtained by applying the well known Little's rule [14].

4.2.2 Expected Transmission Delay of Packets with Second Priority

Using v_1, we obtain the SSPD of the number of packets with second priority to be transmitted in the system at the beginning of the frame. Let us denote $K_{2,n}$ by the number of packets with second priority which succeed in the contention during the n^{th} frame and newly join at the transmission queue. Since a packet has the second priority with probability α_2, the PD of $K_{2,n}$ can be obtained from Equation (13) as follows:

$$\begin{aligned} \Pr(K_{2,n} = l) = k_{2,l} &= \sum_{x=l}^{V} \Pr(K_{2,n} = l | Z = x)\Pr(Z = x) \quad (20) \\ &= \sum_{x=l}^{V} \binom{x}{l} \alpha_2^l (1-\alpha_2)^{x-l} \Pr(Z = x), \quad l = 0, 1, \cdots, V. \end{aligned}$$

Then $X_{n,2}^{m_n}$, the number of packets with second priority in the system at the beginning of the n^{th} frame, when there are m_n packets with first priority, has the following relationship:

$$X_{2,n+1}^{m_1,n+1} = \begin{cases} K_{n,2}, & X_{2,n}^{m_{1,n}} \le \max(0, T - m_{1,n}) \\ X_{2,n}^{m_{1,n}} - (T - m_{1,n}) + K_{2,n}, & X_{2,n}^{m_{1,n}} > \max(0, T - m_{1,n}). \end{cases} \tag{21}$$

Let $\lim_{n\to\infty} X_{2,n}^{m_1,n} = X_2^{m_1}, \lim_{n\to\infty} K_{2,n} = K_2$. It can be shown that $\{X_{2,n}^{m_1,n}, n \ge 1\}$ is a Markov chain process. Then we can obtain the one-step transition probability matrix of this chain as follows:

$$P_{2,m_1} = \begin{pmatrix} k_{2,0} & k_{2,1} & k_{2,2} & \dots & k_{2,T-m+B_2-1} & k_{2,T-m+B_2} \\ k_{2,0} & k_{2,1} & k_{2,2} & \dots & k_{2,T-m+B_2-1} & k_{2,T-m+B_2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ k_{2,0} & k_{2,1} & k_{2,2} & \dots & k_{2,T-m+B_2-1} & k_{2,T-m+B_2} \\ 0 & k_{2,0} & k_{2,1} & \dots & k_{2,T-m+B_2-2} & k^*_{2,T-m+B_2-1} \\ 0 & 0 & k_{2,0} & \dots & k_{2,T-m+B_2-3} & k^*_{2,T-m+B_2-2} \\ 0 & 0 & 0 & \dots & k_{2,T-m+B_2-4} & k^*_{2,T-m+B_2-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & k_{2,T-m-2} & k^*_{2,T-m-1} \\ 0 & 0 & 0 & \dots & k_{2,T-m-1} & k^*_{2,T-m} \end{pmatrix}, k^*_{2,i} = 1 - \sum_{l=0}^{j-1} k_{2,l}. \tag{22}$$

The SSPD, $v_2^{m_1} = \left(v_{2,0}^{m_1}, v_{2,1}^{m_1}, \dots, v_{2,T-m_1+B_2}^{m_1}\right)$, where $v_{2,j}^{m_1} = \Pr(X_2^{m_1} = j)$, of the number of packets with second priority to be transmitted in the system at the beginning of the frame, provided packets with first priority are transmitting, is given by solving the equations:

$$\begin{aligned} v_{2,0}^{(m_1)} &= k_{2,0}\left(\textstyle\sum_{i=0}^{T-m_i} v_{2,i}^{(m_1)}\right), & j \ge 1 \text{ and } \textstyle\sum_{j=0}^{T-m_1+B_1} v_{2,j}^{(m_1)} = 1 \\ v_{2,j}^{(m_1)} &= k_{2,j}\left(\textstyle\sum_{i=0}^{T-m_1} v_{2,i}^{(m_1)}\right) + \textstyle\sum_{l=1}^{j} v_{T-m_1+l^{k}2,j-l}^{(m_1)}, & \text{where } m_1 = 0, 1, \dots, T + B_1. \end{aligned} \tag{23}$$

Using $v_2^{m_1} = (v_{2,0}^{m_1}, v_{2,1}^{m_1}, \dots, v_{2,T-m_1+B_2}^{m_1})$, we obtain the SSPD $v_2 = (v_{2,0}, v_{2,1}, \dots, v_{2,T+B_2})$, where $v_{2,j}$ is the unconditional probability that there are j packets with second priority in the system at the beginning of frame.

$$v_{2,j} = \sum_{m=0}^{T+B_1} v_{2,j}^{m_1} v_{1,m_1}. \tag{24}$$

Then we can calculate the expected number of packets with second priority at the beginning of the frame, that is

$$L_2 = \sum_{j=0}^{T+B_2} j v_{2,j}. \tag{25}$$

The expected transmission delay of packets with second priority in frames is obtained by applying the well-known Little's rule.

4.2.3 Expected Transmission Delay of Packets with i^{th} Priority

In this way, we obtain the SSPD of the number of packets with i^{th} priority to be transmitted in the system at the beginning of the frame. Let us denote $K_{n,i}$ by the number of packets with i^{th} priority which succeed in the contention during the n^{th} frame and newly join at the transmission queue. Since a packet has the i^{th} priority with probability α_i, the PD of $K_{n,i}$ can be obtained from Equation (13) as follows:

$$\begin{aligned} \Pr(K_{n,j} = l) = k_{i,l} &= \sum_{x=l}^{V} \Pr(K_{n,i} = l | Z = x) \Pr(Z = x) \qquad (26) \\ &= \sum_{x=l}^{V} \binom{x}{l} \alpha_i^l (1-\alpha_i)^{x-l} \Pr(Z = x), \quad l = 0, 1, \ldots, V. \end{aligned}$$

Then $X_{n,i}^{m_{i,n},\ldots,m_{i-1,n}}$, the number of packets with i^{th} priority in the system at the beginning of the n^{th} frame, when there are $\sum_{k=1}^{i-1} m_{k,n}$ packets that have higher priority than i^{th} priority, has the following relationship:

$$\begin{aligned} &X_{i,n+1}^{m_{1,n+1},m_{2,n+1},\ldots,m_{i-1,n+1}} \qquad (27) \\ &= \begin{cases} K_{n,i}, & X_{n,i}^{m_{1,n},m_{2,n},\ldots,m_{i-1,n}} \le \max(0, T-a) \\ X_{n,i}^{m_{1,n},m_{2,n},\ldots,m_{i-1,n}} - (T-a) + K_{n,2}, & X_{n,j}^{m_{1,n},m_{2,n},\ldots,m_{i-1,n}} > \max(0, T-a), \end{cases} \end{aligned}$$

where $m_1 = 0, 1, \ldots, T + B_1$.

Let $\lim_{n\to\infty} X_{i,n}^{m_{1,n},\ldots,m_{i-1,n}} = X_i^{m_1,\ldots,m_{i-1}}$, $\lim_{n\to\infty} K_{n,i} = K_i$. It can be shown that $\{X_{n,i}^{m_n}, n \ge 1\}$ is a Markov chain process. Then we can obtain the one-step transition probability matrix of this chain as follows:

$$P_{i,m_1,\ldots,m_{i-1}} = \begin{pmatrix} k_{i,0} & k_{i,1} & k_{i,2} & \ldots & k_{i,T-a+B_i-1} & k_{i,T-a+B_i} \\ k_{i,0} & k_{i,1} & k_{i,2} & \ldots & k_{i,T-a+B_i-1} & k_{i,T-a+B_i} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ k_{i,0} & k_{i,1} & k_{i,2} & \ldots & k_{i,T-a+B_i-1} & k_{i,T-a+B_i} \\ 0 & k_{i,0} & k_{i,1} & \ldots & k_{i,T-a+B_i-2} & k^*_{i,T-a+B_i-1} \\ 0 & 0 & k_{i,0} & \ldots & k_{i,T-a+B_i-3} & k^*_{i,T-a+B_i-2} \\ 0 & 0 & 0 & \ldots & k_{i,T-a+B_i-4} & k^*_{i,T-a+B_i-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & k_{i,T-a-2} & k^*_{i,T-m-1} \\ 0 & 0 & 0 & \ldots & k_{i,T-a-1} & k^*_{i,T-m} \end{pmatrix}, k^*_{i,j}$$

$$= 1 - \sum_{l=0}^{j-1} k_{2,l}. \tag{28}$$

The SSPD, $v_i^{(m_1,\ldots,m_{i-1})} = \left(v_{i,0}^{(m_1,\ldots,m_{i-1})}, \ldots, v_{i,T-m_1+B_2}^{m_1,\ldots,m_{i-1}}\right)$, where $v_{i,j}^{(m_1,\ldots,m_{i-1})} = \Pr\left(X_i^{m_1,\ldots,m_{i-1}} = j\right)$, of the number of packets with second priority to be transmitted in the system at the beginning of the frame, provided m_1 packets with first priority are transmitting, is given by solving the equations:

$$\begin{aligned} &v_{i,0}^{(m_1,\ldots,m_{i-1})} = k_{i,0}\left(\sum_{l=0}^{T-a} v_{i,l}^{(m_1,\ldots,m_{i-1})}\right), \\ &v_{i,j}^{(m_1,\ldots,m_{i-1})} = k_{i,j}\left(\sum_{l=0}^{T-a} v_{i,l}^{(m_1,\ldots,m_{i-1})}\right), + \sum_{l=1}^{j} v_{i,T-a+l}^{(m_1,\ldots,m_{i-1})} k_{i,j-l}, \\ &\quad j \geq 1 \text{ and } \sum_{j=0}^{T-a+B_i} v_{i,j}^{(m_1,\ldots,m_{i-1})} = 1. \end{aligned} \tag{29}$$

Using $v_i^{(m_1,\ldots,m_{i-1})}$, we obtain the SSPD $v_i = (v_{i,0}, v_{i,1}, \ldots, v_{i,T+B_2})$, where $v_{i,j}$ is the un-conditional probability that there are j packets with i^{th} priority in the system at the beginning of frame.

$$v_{i,j} = \sum_{m_1=0}^{T+B_1} \sum_{m_2}^{Max(0,T-m_1)+B_2} \cdots \sum_{m_{i-1}=0}^{\max(0,T-a)+B_{i-1}} v_{i,j}^{(m_1,\ldots,m_{i-1})} v_{i-1,m_{i-1}}^{(m_1,\ldots,m_{i-2})}, \ldots, v_{2,m_2}^{(m_1)} v_{1,m_1}. \tag{30}$$

Then we can calculate the expected number of packets with i^{th} priority at the beginning of the frame, that is

$$\sum_{j=0}^{T+B_2} j v_{i,j}. \tag{31}$$

The expected transmission delay of packets with i^{th} priority in frames is obtained by applying the well known Little's rule.

5 NUMERICAL RESULTS

In numerical results to verify our proposed approach, we compute the expected delays (i.e., access and transmission delay) for voice and data call messages using the parameters $V = 30$, $T = 95$, $B = 10$, $\lambda = 5$, $\alpha = 0.3$; the expected number of packets of a data call message is set to be 1 000 (i.e., $\delta = 0.001$). Expected delays are calculated as the expected number of packets in a voice call message (i.e., $1/\varepsilon$) varies. The same was found using simulations as well. Figure 5 shows close agreement between analytical results and the simulation, especially in the voice call message, thus validating the analysis. As for the data call message, however, analytical results tend to overestimate the simulation results (but are still within the confidence intervals). This is because we first derived the probability distribution of the number of data call packets in the system under the condition that there are k voice call messages transmitting, and then unconditioned it. This is based on the assumption that the number of data call packets reaches steady state before k changes.

6 CONCLUSION

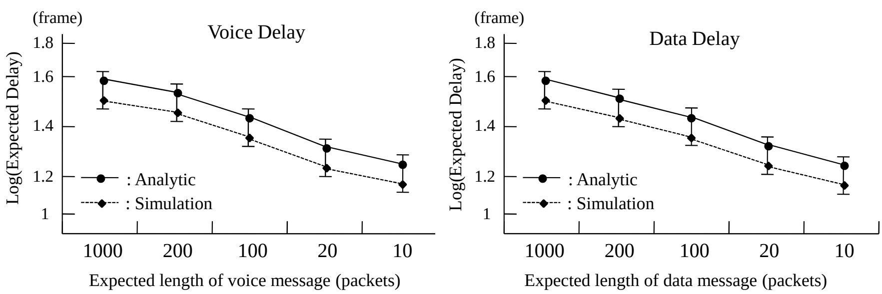

Fig. 5. Expected delays

We described the analytical model of the BEB policy with multiple priority calls, which is a collision resolution algorithm often adopted in the random access packet mobile networks. We obtain the SSD of the number of messages waiting in mobile information systems, which is utilized to get the probability that a tagged message experiences a collision given that it actually participates in the contention for a request slot in a frame, which has never been investigated in the literature.

With this model, the expected access delay and expected transmission delay of packets that a call message experiences from its arrival to the mobile system until the successful transmission are found analytically. In numerical results, we computed the expected (i.e., access and transmission) delays for voice and data call messages using the parameters. The proposed analytical model compared the performance indexes computed with simulation results. It showed that our analytic model gives an excellent agreement with the simulation results.

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