Computing and Informatics, Vol. 28, 2009, 339–351

ADAPTIVE OPTIMAL DYNAMIC CONTROL FOR NONHOLONOMIC SYSTEMS

József K. Tar, Imre J. Rudas

Institute of Intelligent Engineering Systems
John von Neumann Faculty of Informatics
Budapest Tech Polytechnical Institution
Bécsi út 96/B
H-1034 Budapest, Hungary
e-mail: tar.jozsef@nik.bmf.hu, rudas@bmf.hu

Manuscript received 3 April 2008; revised 18 July 2008
Communicated by Stanislava Labátová

Abstract. In this paper two different control methods are combined for controlling a typical nonholonomic device (a bicycle) the dynamic model and parameters of which are only approximately known. Most of such devices suffer from the problem that the time-derivatives of the coordinates of their location and orientation cannot independently be set so an arbitrarily prescribed trajectory cannot precisely be traced by them. For tackling this difficulty *Optimal Control* is proposed that can find acceptable compromise between the tracking error of the various coordinates. Further problem is that the solution proposed by the optimal controller cannot exactly be implemented in the lack of precise information on the dynamic model of the system. Based on the decoupled nature of the dynamic model of the longitudinal and lateral behavior of the engine special fixed point transformations are proposed to achieve adaptive tracking. These transformations were formerly successfully applied for the control of holonomic systems. It is the first time that the combined method is checked for various trajectories and dynamic model errors via simulation. It yielded promising results.

Keywords: Nonholonomic systems, optimal control, adaptive control, fixed point transformations

1 INTRODUCTION

Due to the fact that the Cartesian coordinates and the orientation (i.e. rotational angle) of certain most commonly used wheeled vehicles cannot independently be prescribed two typical motion regimes have to be distinguished: a) iterative, small, "back and forth" type movements for leaving a place/occupying a vacancy in a crowded parking place with the aim of considerably modifying the rotational angle of the vehicle at the cost of minimal modification of its position, and b) tracking control of a path of considerable velocity. For prescribing *realizable nominal paths* the trajectory to be tracked normally should be built up by keeping in mind the kinematic model of the vehicle that yields local trajectory data in such terms as its local curvature, local center of rotation, and in general by using various quantities defined by the Frenet frame locally fitted to the path.

Optimal control methods found applications in various practical fields (e.g. [1]). As alternative approach to solving the tracking problem kinematically typically good solution may be the application of optimal control under strict constraints in which a compromise can be found between various controversial requirements as e.g. simultaneously required pose and position. The constraints must be taken into account precisely because they guarantee the physical interpretability and realizability of the computed results, while the position and pose tracking can only be (well) approximated. In this manner a lot of complicated computations can be evaded and replaced by far simpler ones.

It is important to note that this planning problem cannot be satisfactorily realized by completely omitting the dynamic model of the vehicle under control. Subsequently incorrect realization of the kinematically designed trajectory data may lead to accumulation of the errors and may result in wrong proposed proposition by the optimal control. The lateral control of vehicle systems obtained considerable attention in the recent literature, e.g. [2, 4]. The effects of the actuator dynamics of the steering system were described in the literature by a simplified vehicle model with good results (e.g. [3, 4]). In the approach presented here the mathematical structure of this is utilized but its parameters are assumed to be known only imprecisely.

Instead of applying Pontryagin's original method that can express the kinematic restrictions as constraints with associated Lagrange multipliers in the proposed solution these restrictions are explicitly utilized so the simplest form of the *Reduced Gradient Method* (i.e. the simple *Gradient Descent* without any gradient reduction) becomes applicable. Simulation calculations using the MS Excel's Solver package with Visual Basic taking into account the dynamics of the steering wheel and that of the vehicle in the longitudinal direction are presented to show the applicability of this concept even in the case of approximate knowledge of the dynamic parameters. However, for more precise tracking some adaptive control is needed.

Heartened by the success of various geometric approaches in adaptive control (e.g. [5, 6]) elaborated at Budapest Tech, in the present paper a novel, two parametric fixed point transformation based adaptive control is applied. The idea of this new variant was published in [7].

The paper is organized as follows: at first the kinematic and dynamic models of the paradigm used (i.e. the bycicle) are presented and substantiated. Following that the "otrhodox" and "non orthodox" applications of *Pontryagin's Optimal Control* approach are briefly discussed. The next brief part is devoted to the idea of the adaptive control applied. Finally simulation examples are presented and conclusions are drawn.

2 THE KINEMATIC AND DYNAMIC MODELS OF THE BICYCLE

The simple device called bicycle is a good paradigm for a wide set of vehicles. Its control variables are the velocity of the rear end v in m/s, and the rotational angle of the steering wheel δ in rad units. With respect to a properly chosen local Frenet frame the motion can temporarily be regarded as a rigid rotation around a fictitious temporal center point. The front and the rear wheels are situated at different distances from the local center of rotation; therefore they move at different speeds. If Φ denotes the rotational pose of the body of the bicycle while x and y are the Cartesian coordinates of the rear end (wheel center), the kinematic relationships are given by (1):

$$\dot{x} = v\cos(\Phi), \dot{y} = v\sin(\Phi), \dot{\Phi} = v\frac{\tan(\delta)}{L}. \tag{1}$$

For tracking a smooth trajectory it would be convenient to prescribe the *"nominal velocities"* as $\dot{x}^N$, $\dot{y}^N$, and independently of them the rotational velocity of the body, $\dot{\Phi}^N$. For instance, it could be expedient to require Φ to be equal to the angle of the tangent of the trajectory. However, (1) makes it clear that at an actual pose Φ the prescribed $\dot{x}$ determines v and $\dot{y}$, while the prescribed $\dot{\Phi}$ and v determine δ, therefore it is impossible to track a path consisting of the independently given three velocities. It is also clear that instead of exactly determining two of the three different data and completely ignoring the third one finding a compromise allowing some tracking error "distributed" between these three variables may be reasonable. For this purpose minimization of a cost function composed of these three error terms plus other optional terms can be expedient. This task will be considered in the section describing the optimal control.

Regarding the dynamic model of the bicycle, according to [3], [4] the lateral and the longitudinal behavior seem to be decoupled as

$$\dot{\delta} = -\frac{\delta}{\tau} + \frac{K_a}{\tau}u, \dot{v} = \frac{M_{mod}}{M}\dot{v}^{Des} - \frac{\mu}{M}v \tag{2}$$

where M is the mass of the whole system, M_{mod} is its model value, μ is a viscous friction coefficient of the driving wheel, $\dot{v}^{Des}$ is the desired acceleration. Regarding the dynamics of the steering wheel in (2) it is supposed that the momentum and the rotational acceleration of the wheel can be neglected, τ is the time constant of its drive, u denotes the control signal, and K_a is some gain factor. While the dynamic model of the driving wheel is physically well substantiated by Newton's 2$^{\text{nd}}$ Law, that of the steering wheel needs some substantiation. For this purpose consider the dynamic model of a damped system: $m\ddot{x} + \mu\dot{x} = f$ in which the inertia m is very small, and f denotes the external exciting force. Integration of this equation according to time leads to a 1st order inhomogeneous differential equation $m\dot{x}+\mu x = F_0 + \int_0^t f(\xi)d\xi \equiv F(t)$. The homogeneous part of this equation evidently has the solution $x_{Hom}(t) = C\exp\left(-\frac{\mu t}{m}\right)$ of which a particular solution of the inhomogeneous one can be built in by making the coefficient C vary in time. The general solution is $x_{Inh}(t) = \exp\left(-\frac{\mu t}{m}\right)\left[C(0) + \int_0^t d\xi F(\xi)\exp\left(\frac{\mu\xi}{m}\right)/m\right]$. For constant excitation f_c in the $t \to \infty$ limit we obtain that $x_{Inh} = \frac{F_0}{\mu} - \frac{mf_c}{\mu^2} + \frac{f_c t}{\mu} + transients$ where the transients are damped by the exponent $-\frac{\mu}{m}$. In this limit evidently $\dot{x}_{Inh} \approx f_c/\mu$, that is the solution of the equation without any inertia is obtained: $\mu\dot{x} = f_c$. This simple case study substantiates the omission of the momentum of the steering wheel in our model. In the next section the idea of optimal control is briefly summarized.

3 THE IDEA OF OPTIMAL CONTROL

The essence of any optimal control method is the minimization of some cost function that is constructed as a weighted sum of nonnegative contributions. Each of these contributions expresses some error term that takes the value of zero when no error is present, and yields positive value for finite error. The weighting factors determine the nature of the "compromise" between the error terms when it is impossible to drive each error term simultaneously to zero. For solving such tasks many efficient solutions, e.g. the simple *Gradient Descent Method (GDM)* applicable for differentiable cost functions is a plausible choice. A different class of optimization problems is formed by the tasks in which the optimum (either minimum or maximum) of a function $f(\vec{x})$ has to be found *under constraints* normally expressed in canonical form as $\{g^{(i)}(\vec{x}) = 0 | i = 1, \ldots, K < N\}$, $\vec{x} \in \Re^N$. This task was solved first by Lagrange in relation with Classical Mechanical Problems [10] for differentiable functions according to the following philosophy: after founding a common point of the constraint surfaces, instead of moving in the direction of the steepest variation of f (i.e. in the direction of $\vec{\nabla} f$) the next point has to be found in the direction of its reduced form $\widetilde{\vec{\nabla} f} := \vec{\nabla} f + \sum_{s=1}^{K} \lambda_s \vec{\nabla} g^{(s)}$ where the $\{\lambda_s\}$ Lagrange multipliers must be so determined that $\widetilde{\vec{\nabla} f}$ must be orthogonal to each $\vec{\nabla} g^{(s)}$. (In this manner in the first order approximation the next point also remains on the constraint surface.) When the reduced gradient achieves the value zero the local solution is found. This witty idea was utilized by Pontryagin in his *Optimal Control* [1].

The idea of optimal control was developed for dynamic systems of state $\vec{x}$ having the equation of motion $\dot{\vec{x}} = \vec{f}(\vec{x}, \vec{u})$ where $\vec{u}$ denotes the control signal. If a cost function $J(\vec{x}, \dot{\vec{x}}, \vec{u}) \equiv \tilde{J}(\vec{x}, \vec{u})$ is given, its integral has to be minimized in the interval $[0, T]$. With a small Δt time-resolution this integral can be approximated as the sum $\sum_{s=0}^{N} \tilde{J}(\vec{x}_s, \vec{u}_s)\Delta t$ under the constraints $[\vec{x}_{s+1} - \vec{x}_s]/\Delta t \approx \vec{f}(\vec{x}_s, \vec{u}_s)$. Since $\Delta t > 0$ we can minimize the sum $\sum_{s=0}^{N} \tilde{J}(\vec{x}_s, \vec{u}_s)$ under the same constraints. In the finite element approximation the variables by the variation of which the cost can be minimized are comprised in the sets $\{\vec{x}_s|s = 1, \ldots, N\}, \{\vec{u}_s|s = 0, \ldots, N-1\}$ ($\vec{x}_0$ corresponds to the initial condition). This problem can be solved by Lagrange's *Reduced Gradient Method* in which with each constraint variable a Lagrange multiplier is associated in the *"adjoint task"*, i.e. constraint-free minimization/maximization of a function $\Psi(\{\vec{x}\}, \{\vec{u}\}, \{\vec{\lambda}\}) = \sum_{s=0}^{N} \tilde{J}_s + \sum_{s=0}^{N-1} \vec{\lambda}_s^T [\frac{\vec{x}_{s+1}-\vec{x}_s}{\Delta t} - \vec{f}_s]$ is needed. The derivatives of Ψ according to the components of $\vec{\lambda}_s$ yield the constraints. The derivatives according to the $\vec{x}_k$ components yield that $\frac{\partial(\tilde{J}_k - \vec{\lambda}_k^T \vec{f}_k)}{\partial \vec{x}_k} + \frac{\vec{\lambda}_{k-1}-\vec{\lambda}_k}{\Delta t} = 0$, and finally, the partial derivatives according to the $\vec{u}_k$ components imply that $\frac{\partial(\tilde{J}_k - \vec{\lambda}_k \vec{f}_k)}{\partial \vec{u}_k} = 0$. Turning back to the "continuous limit" this leads to typical *"canonical equations of motion"* with a "Hamiltonian" $H := \vec{\lambda}^T \vec{f} - \tilde{J}$ as

$$\dot{\vec{\lambda}} = -\frac{\partial H}{\partial \vec{x}}, \dot{\vec{x}} = \frac{\partial H}{\partial \vec{\lambda}}, \frac{\partial H}{\partial \vec{u}} = 0. \tag{3}$$

It is worth noting that (3) is *not completely equivalent with the solution of the original problem* due to various reasons:

1. the same canonical equations belong to the maximum and the minimum problems;
2. in (3) the initial values of the variables $\vec{\lambda}$ does not seem to be determined though the initial values of the Lagrange–multipliers are determined by a previous optimum search.

So the canonical equations have rather "symbolic" than practical significance in the case of these tasks. They are important from a special point of view: they canonically map the state space onto itself in time with a symplectic Jacobian. The spectrum of the symplectic matrices has the following special properties (e.g. [11]): if χ is an eigenvalue of a symplectic matrix then $\bar{\chi}$, $1/\chi$, and $1/\bar{\chi}$ [the $\bar{\ }$ symbol denotes the complex conjugate] are eigenvalues, too. This implies that in normally numerical problems instabilities have to arise while solving (3). The best case belongs to the $|\chi| = 1$ eigenvalues that may be also dubious due to the limited precision of the digital representation of the numbers. This emphasizes the significance of finding the actual minimum of Ψ by correctly applying the *Reduced Gradient Method*.

As will be shown later the application of the *Lagrange Multipliers* in such tasks can be avoided so the task of searching the optimum can be formulated as a simple *Steepest Descent Method* without any reduction of the gradients. For this purpose the optimum can be found only for a single step in the control cycle and certain values in the "next time" need not be varied: instead, they can be estimated from the velocities. In the finite element approximation scheme this choice corresponds to the estimation of the time-derivative of a function h as $\dot{h}(t_{k+1}) \approx [h(t_{k+1}) - h(t_k)]/\Delta t$ instead of the other plausible possibility for the estimation as $\dot{h}(t_k) \approx [h(t_{k+1}) - h(t_k)]/\Delta t$. The lack of the need of using Lagrange multipliers makes it possible to apply alternative optimization methods, like *Particle Swarm Optimization (PSO)* [12], too.

In the sequel the basic idea of the adaptive control is briefed that "studies" the controlled system according to an "excitation–response" scheme.

4 THE IDEA OF THE NOVEL ADAPTIVE CONTROL: THE EXCITATION – RESPONSE SCHEME COMBINED WITH FIXED–POINT TRANSFORMATION

Each control task can be formulated by using the concepts of the appropriate *"excitation"* Q of the controlled system to which it is expected to respond by some prescribed or *"desired response"* r^d. The appropriate excitation can be computed by the use of some *inverse dynamic model* $Q = \varphi(r^d)$. Since normally this inverse model is neither complete nor exact, the actual response determined by the system's dynamics, ψ, results in a *realized response* r^r that differs from the desired one: $r^r \equiv \psi(\varphi(r^d)) \equiv f(r^d) \neq r^d$. It is worth noting that the functions $\varphi()$ and $\psi()$ may contain various hidden parameters that partly correspond to the dynamic model of the system, and partly pertain to unknown external dynamic forces acting on it. Due to phenomenological reasons the controller can manipulate or "deform" the input value from r^d so that $r^d \equiv \psi(r^d_*)$. Other possibility is the manipulation of the output of the rough model as $r^d \equiv \psi(\varphi^*(r^d))$. In the sequel it will be shown that for *SISO* systems the appropriate deformation can be defined as some *Parametric Fixed Point Transformation.* For this purpose consider the simple iteration described by (4) that is suggested by the presence of similar triangles in a very simple drawing [7]:

$$\begin{gathered}
g(x|x^d, D_-, \Delta_+) := \\
= \frac{(f(x)-\Delta_+)(x-D_-)}{x^d-\Delta_+} + D_-, \\
if \quad f(x_\star) = x^d \quad then \quad g(x_\star) = x_\star, \\
g' = f'(x)\frac{x-D_-}{x^d-\Delta_+} + \frac{f(x)-\Delta_+}{x^d-\Delta_+}, \\
g'(x_\star|x^d, D_-, \Delta_+) = 1 + f'(x_\star)\frac{x_\star-D_-}{x^d-\Delta_+}.
\end{gathered} \tag{4}$$

According to (4) it is evident that the requested solution $x_\star$ just is the fixed–point of the function g, and that by appropriately choosing the initial model and the control parameters D_-, and Δ_+ the condition of $|dg/dx| \leq K < 1$ can be achieved in a vicinity of $x_\star$. This region around $x_\star$ serves as a basin of attraction for the iteration $x_{n+1} = g(x_n)$ since if a and b are within this region then $|g(a) - g(b)| = |\int_a^b g' dx| \leq \int_a^b |g'| dx \leq K|a - b|$, i.e. $g()$ realizes a *contractive mapping* resulting in a *Cauchy Sequence* in the complete normed linear space of the real numbers with the norm defined by the absolute value. Really, for arbitrary natural number L it can be stated that

$$\begin{gathered}
|x_{n+L} - x_n| = |g(x_{n+L-1}) - g(x_{n-1})| \leq \dots \\
\leq K^n |x_L - x_0| \to 0 \;\; as \quad n \to \infty,
\end{gathered} \tag{5}$$

therefore the sequence $\{x_n\}$ must converge to a limit value c. This limit value evidently must be equal to the fixed point $x_\star$ since $|g(c)-c| \equiv |(g(c)-x_n)+(x_n-c)| \leq |g(c)-x_n|+|x_n-c| = |g(c)-g(x_{n-1})|+|x_n-c| \leq K|c-x_{n-1}|+|x_n-c| \to 0$ as $n \to \infty$. For utilizing this convergence in the adaptive control it is just enough to prescribe some appropriate *desired response* on the basis of simple kinematical considerations. If the variation of x^d in time is far slower than the speed of convergence of the above iteration, the idea of *Complete Stability* can similarly be applied as e.g. in the case of fast real-time image processing [8], and a good adaptive tracking control can be achieved. (More precisely, within one control cycle only one step of iteration can be executed.) This expectation is also supported by the results of previous investigations made in connection with less lucid fixed–point transformations [6]. In the sequel this idea will be used in the proposed adaptive control of the bicycle. It has to be noted that in its philosophical background this idea is smilar to the idea of "situational control" [9].

5 ADAPTIVE TRACKING CONTROL FOR THE BICYCLE

Let us suppose that the actual x, y, Φ, v, and δ values are given as *"initial conditions"* in a given time instant, and we seek the "next" values using finite time-resolution Δt. According to the kinematic model in (1) we must have the following values: $x^{Next} = x + \Delta t \cdot \cos(\Phi^{Next})$, $y^{Next} = y + \Delta t \cdot \sin(\Phi^{Next})$, $\Phi^{Next} = \Phi + \Delta t \cdot v^{Next} \tan(\delta^{Next})/L$, v^{Next}, and δ^{Next}. If we also have the nominal values for the "next" time as x^N, y^N, Φ^N, a cost function with positive A, B, and C coefficients can be constructed as

$$\tilde{J} := A\sqrt{(x^N - x^{Next})^2 + (y^N - y^{Next})^2} + B|\Phi^N - \Phi^{Next}| + C(\delta^{Next})^8. \qquad (6)$$

The term with the coefficient A "prohibits" the tracking error, the term with factor B stands for the reduction of the error in the angular pose, the term with factor C inhibits the application of "extreme" steering wheel angles. This cost function evidently takes into account the "strict constraints" without the application of any Lagrange-multiplier and can be minimized according to two variables δ^{Next} and Φ^{Next}. (Naturally many other possibilities exist for the construction of various cost functions.) The application of the $|\bullet|$ function leads to more sensitive gradients around 0 than the $(\bullet)^2$ function and has problems with the differential only in the point of 0. On this reason certain terms are formulated by the use of the $|\bullet|$ function, and the application of the $\surd$ plays similar role. The only exception is the value of the steering angle δ that needs small prohibition for small values and quite drastic prohibition for bigger angles. It is evident that (6) has the minimum, and by setting the A, B, and C coefficients the compromise between the requirements in contradiction can be manipulated. In the "ideal case" in which no dynamic effects are taken into account the result of the optimization really corresponds to the "next point". If the effects of the system dynamics are also taken into consideration the optimization yields only some "desired next values" that according to the dynamic model in (2) are related to the "actual next values" v_{Next}^{Act} as

$$\frac{M_{mod}}{M}\dot{v}^{Des} - \frac{\mu}{M}v_{Next}^{Act} = \frac{v_{Next}^{Act} - v}{\Delta t} \equiv \dot{v}^{Act} \tag{7}$$

in which $\dot{v}^{Des} = (v^{Next} - v)/\Delta t$ contains the result of the optimization. In similar manner $\dot{\delta}^{Act} = \frac{K_a \cdot (\delta^{Des} - \delta) - \delta}{\tau}$ if the control signal for the steering angle is a simple error feedback as $u \equiv (\delta^{Des} - \delta)$. Since (7) can analytically be solved for v_{Next}^{Act} it is very easy to develop a simulation program for the description of the control of the bicycle by the use of MS EXCEL and Visual Basic. This software has excellent properties and can efficiently be used in the higher education (see e.g. [13, 14]). Certain functional relationships can be coded within the worksheets as well as the appropriate settings or "model" for the SOLVER package. The Visual Basic program in this case can be reduced to filling in certain cells in the worksheets, executing the optimization by the simple commands as *SolverSolve UserFinish := True*, *SolverFinish KeepFinal := 1*, reading the results of optimization and store the simulation data in a worksheet for convenient construction of graphs. Before going into the cycle the SOLVER's model settings can be read from the appropriate worksheet by the command line as e.g. *SolverLoad LoadArea := Range("B27:B29")*. The implementation of the adaptive control independently for δ and v is also very easy. In the sequel simulation results will be presented.

6 COMPUTATIONAL RESULTS

Various cases were investigated. The *"ideal case"* corresponded to pure kinematic considerations, i.e. in this case it was supposed that the results of the kinematic trajectory tracking were exactly realized. The next case was the *"realistic case"* in which the dynamics of the system was taken into account without applying any adaptivity. Since it was found that the main role in the adaptive control was played by the loop adjusting v while the adaptive loop for δ modified only nuances on its resulst, we can use the notation *"adaptive case"* for the full adaptivity.

Due to the periodic nature of the sin, cos, and tan functions for $\pm 2\pi$ in (1) special care was needed for prescribing the "next nominal value" Φ_{Next}^{N}. For realizable motion it must be in the vicinity of the previously prescribed one (big $\pm\Phi$ values correspond to multiple rotations) while calculating Φ from the arctan function gives a value only in the $[-\pi, \pi]$ interval. Furthermore, since $\sin(\Phi \pm \pi) = -\sin(\Phi)$ and $\cos(\Phi \pm \pi) = -\cos(\Phi)$ and $\tan(-\delta) = -\tan(\delta)$ the simultaneous transformations as $\Phi \to \Phi \pm \pi$, $v \to -v$, and $\delta \to -\delta \pm 2\pi$ kinematically correspond to the same result but in the consecutive steps these cases dynamically are not equivalent with each other. However, since the SOLVER can find local minima/maxima one must be careful with periodic cost functions. For solving these problems in the cost function (6) the following values were chosen: $A = 500$, $B = 50$, and $C = 1$. This small C coefficient well kept the $|\delta|$ values at bay through the term $C\delta^8$. For correctly tracking the Φ^N_{Next} value the result of the arctan function was increased/decreased with the value 2π until it has been transformed to the vicinity of the previous step. Another important factor is the ratio of the length of the bicycle body ($L = 0.5$ m was used in the simulations) and the size of the "sharp curved parts" in the trajectory as well as the structure of the cost function (6) that may be built up by using various different increasing functions, too. The dynamic parameters of the system considered were as follows: the actual inertia $M = 2$ kg, its model value $M_{mod} = 1$ kg, $\tau = 0.095\ s$ time-constant of the steering wheel's dynamics, $\mu = 10$ Ns/m, while in the adaptive control $\Delta^\delta_+ = 2 \cdot 10^6$, $D^\delta_- = 1 \cdot 10^6$, $\Delta^v_+ = 2500$, $D^v_- = -3500$. The time-resolution of the calculations was $\Delta t = 0.05$ s. In Figure 1 typical results are given for tracking a circular nominal trajectory in the "ideal" and the "fully adaptive" cases. Apart from the initial transients of the adaptive control the differences seem to be only nuances. Figure 2 reveals more details that substantiate this statement. (The circular trajectory is trivially a special one that could exactly be traced by such devices.)

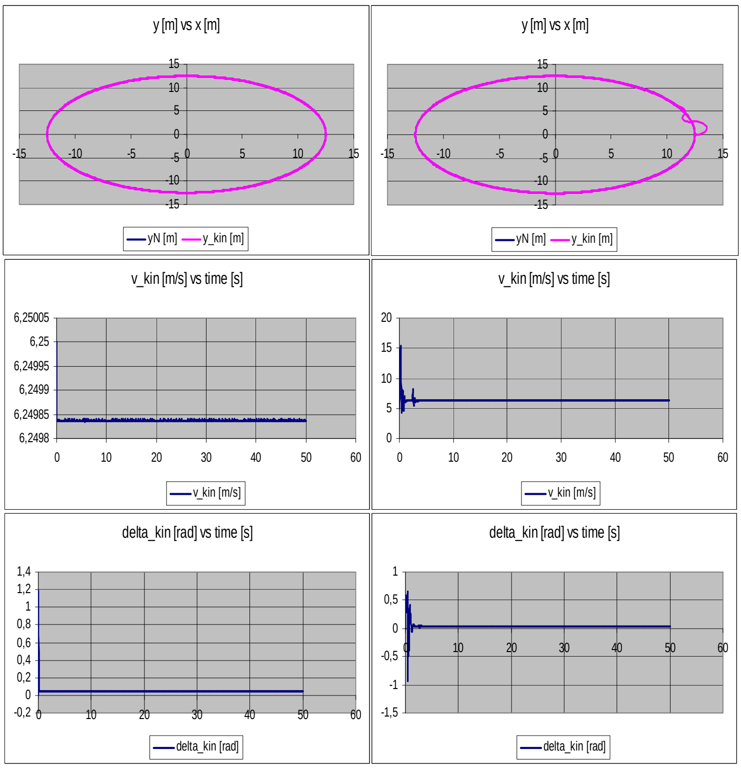


Fig. 1. Comparison of the "ideal" (LHS) and the "fully adaptive" (RHS) cases: the trajectory (1st line), the velocity of the bicycle (2nd line), and the steering angle (3rd line)

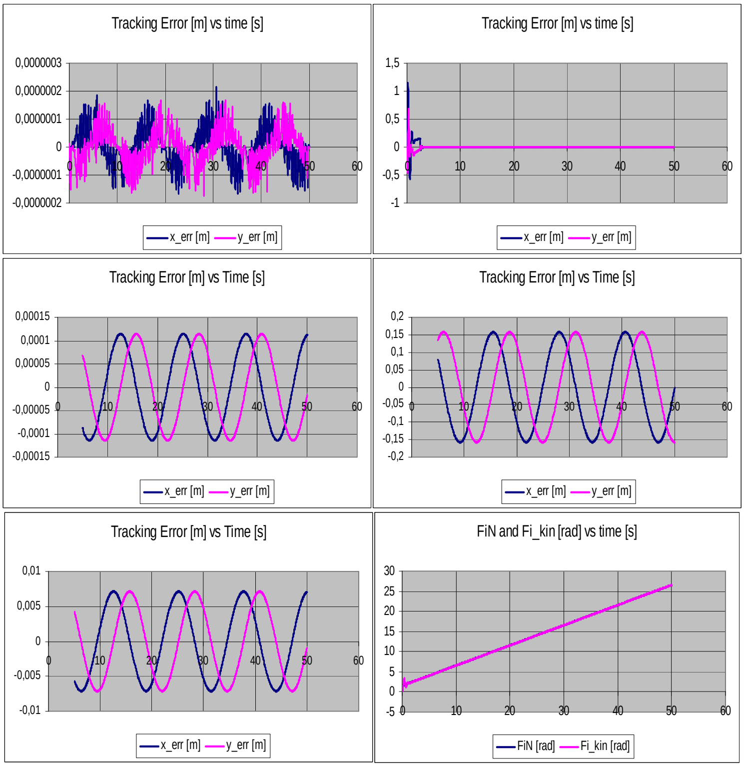


Fig. 2. Tracking errors of the "ideal" (upper left), the "fully adaptive" (upper right) cases, fine details of the tracking error for the "fully adaptive" and the "non-adaptive" cases (middle line), tracking error when only the v loop is adaptive (lower left), and the pose tracking for the "fully adaptive" case (lower right chart)

Tracking of an interesting trajectory can be seen in Figure 3. The shape of number 8 is not a smooth line in it. It is rather constructed of letter 3 and its mirrored version. This means that v and δ has to change sign, that is the vehicle stops two times in the center. Certain results are given in Figure 3. It is worth noting that the "stabilized amplitude" of the tracking error on the nonadaptive case is a little bit greater than $\pm 0.2\ m$ that exemplifies the operation of the adaptive loops.

We note that similar stable results were obtained for more or less smooth nominal trajectories of various shapes.

7 CONCLUSIONS

In this paper a combined application of the Optimal Control and a fixed point transformations based adaptive control was presented to control a dynamically only

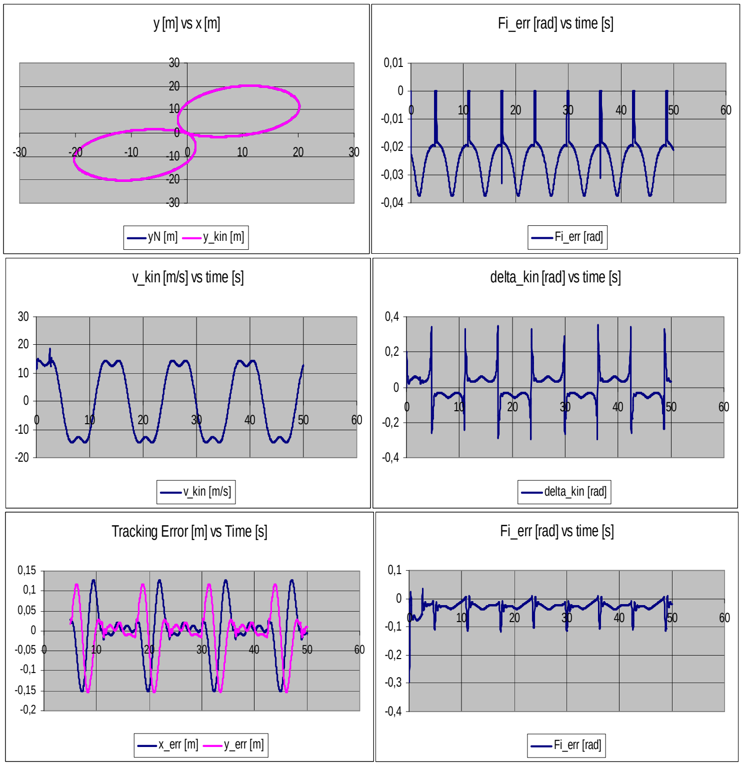


Fig. 3. Tracking an 8 shaped trajectory and pose in the "ideal" case (upper line), the velocity and the steering angle in the "fully adaptive" case (central line), fine details of the tracking and the pose error for the "fully adaptive" case (lower line)

approximately known device, the bicycle, that serves as a good paradigm of a wide set of nonholonomic devices. The optimal control was to resolve the conflict between the arbitrarily prescribed nominal trajectory and the kinematical abilities of this device. The adaptive part's function was to precisely realize the kinematically optimal motion in spite of the modeling errors in the dynamics. It was found that the problems originating from the periodic nature of the cost function applied can successfully be tackled. It is expected that choosing various cost functions further solutions can be obtained.

Acknowledgment

The authors gratefully acknowledge the support by the Hungarian National Research Fund (OTKA) within the project No. K063405. This research was also supported by the National Office for Research and Technology (NKTH) and Agency for Research Fund Management and Research Exploitation (KPI) in Hungary using the resources of the Research and Technology Innovation Fund within the project No. RET-10/2006.

REFERENCES

[1] Bryson, E. Jr.—Yu-Chi, H.: Applied Optimal Control. Hemisphere, 1975.

[2] Suárez, J. I.—Vinagre, B. M.: A Proposal for Parameter Tuning in Fractional MRAC. Application to the Lateral Control of an Autonomous Vehicle, Preprints of the 6th IFAC Symposium on Intelligent Autonomous Vehicles (IAV 2007), LAAS-CNRS, Toulouse, France, September 3–5, 2007.

[3] Rodriguez-Castaño, A.—Ollero, A.—Vinagre, B. M.—Chen, Y.: Fractional Controller for Guidance of Autonomous Ground Vehicles. Preprints 5th IFAC International Symposium on Intelligent Components and Instruments for Control Applications – SICICA 2003, pp. 97–100, July, 2003.

[4] Heredia, G.—Ollero, A.—Gordillo, F.—Aracil, J.: Stability Analysis of Fuzzy Path Tracking Using a MIMO Frequency Response Technique. In A. Ollero (ed.), Preprints of the IFAC Workshop on "Intelligent Components for Vehicles", pp. 15–20, 1998.

[5] Tar, J. K.—Rudas, I. J.—Bitó, J. F.—Tenreiro Machado, J. A.: Centralized and Decentralized Applications of a Novel Adaptive Control. In Proc. of the 9th International Conference on Intelligent Engineering Systems (INES 2005), September 16–19, 2005, "Cruising on Mediterranean Sea, File: `Tar.pdf` (CD issue)".

[6] Tar, J. K.—Rudas, I. J.—Szeghegyi, Á.—Kozłowski,K.: Novel Adaptive Control of Partially Modeled Dynamic Systems. In: Lecture Notes in Control and Information Sciences – Robot Motion and Control: Recent Development, Part II – Control and Mechanical Systems, Springer Berlin/Heidelberg, 2006, Ed. K. Kozłowski.

[7] Tar, J. K.—Rudas, I. J.—Kozłowski, K. R.: Fixed Point Transformations-Based Approach in Adaptive Control of Smooth Systems. Lecture Notes in Control and Information Sciences – Robot Motion and Control 2007 (Ed. K. Kozłowski), Springer-Verlag London Ltd., June 11–13, 2007, Bukowy Dworek, Poland, pp. 157–166.

[8] Roska, T.: Development of Kilo Real-time Frame Rate TeraOPS Computational Capacity Topographic Microprocessors. Plenary speech at the 10th International Conference on Advanced Robotics (ICAR 2001), August 22–25, 2001, Budapest, Hungary.

[9] Andoga, R.—Madarász, L.—Főző, L: Situational Modeling and Control of a Small Turbojet Engine MPM 20. Proc. of the IEEE International Conference on Computational Cybernetics, August 20–22, 2006, Tallinn, Estonia, pp. 81–85.

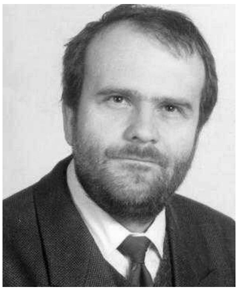
[10] Lagrange, J. L.: Mécanique Analytique (Analytical Mechanics). 4. ed., 2 vols. Paris: Gauthier-Villars et fils, 1888–89, First Edition: 1788.

[11] Arnold, V. I.: A Mechanika Matematikai Módszerei. (Mathematical Methods of Classical Mechanics.) Műszaki Könyvkiadó, Budapest, 1985.

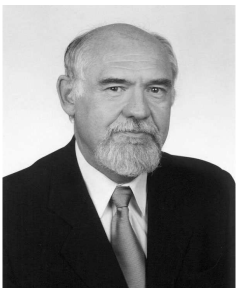
[12] Kennedy, J.—Eberhart, R.: Particle Swarm Optimization. Proc. of IEEE Intl. Conf. on Neural Networks, Perth, pp. 1942–1948, 1995.

[13] ŐRI—I.—KISS, G.: Teaching Probability Theory and Mathematical Statistics Using Microsoft Excel. In the Proc. of the 3rd International Conference on Information Technology Based Higher Education and Training (ITHET 02) (CD issue), Budapest, Hungary, July 4–6, 2002.

[14] TAR, J. K.—SZEGHEGYI, Á.: The Use of MS Office and Intel Pentium Processors-based PCs in Teaching Thermodynamics. In the Proc. of the 3rd International Conference on Information Technology Based Higher Education and Training (ITHET 02) (CD issue), Budapest, Hungary, July 4–6, 2002.



József K. TAR Ph. D. IEEE Member graduated as physicist at Eötvös Lóránd University, Budapest, Hungary in 1981. From 1981 to 1984 he worked for Tungsram Co. Ltd. – Central Research Institute for Physics, Budapest, in a far infrared laser development project. From 1985 to 1993 he had various positions at Tungsram Co. Ltd. From 1993 he is involved in education at Budapest Tech and its legal predecessor. His research area includes soft computing, robotics, and nonlinear control. He has about 9 book chapters, 37 journal publications, 131 papers in conference proceedings.



Imre J. RUDAS from 1971 to 1976 worked as Engineer at United Electric-Machine Company, and later at the Bánki Donát Polytechnic, Budapest, Hungary. From 1976 to 1990 he had various educational responsibilities at the Bánki Donát Polytechnic, became full professor in 1990. From 2000 to 2003 he worked as the Vice President of Budapest Tech Polytechnical Institution, Budapest, Hungary. From 2003 he works as the Rector/President of Budapest Tech. His research area includes fuzzy systems, robotics, and nonlinear control. He has 28 book chapters, 56 journal publications, 280 papers in conference proceedings.