MORE EFFICIENT ON-THE-FLY VERIFICATION METHODS OF COLORED PETRI NETS

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Abstract. Colored Petri Nets (CP-nets or CPNs) are powerful modeling language for concurrent systems. As for CPNs' model checking, the mainstream method is unfolding that transforms a CPN into an equivalent P/T net. However the equivalent P/T net tends to be too enormous to be handled. As for checking CPN models without unfolding, we present three practical on-the-fly verification methods which are all focused on how to make state space generation more efficient. The first one is a basic one, based on a standard state space generation algorithm, but its efficiency is low. The second one is an overall improvement of the first. The third one sacrifices some applicability for higher efficiency. We implemented the three algorithms and validated great efficiency of latter two algorithms through experimental results.

Keywords: Model checking, CPN, on-the-fly, LTL, state space

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1 INTRODUCTION

CPNs are powerful graphical language for modeling concurrent systems introduced by Jensen in 1981 [9]. As a kind of high-level Petri nets, CPN is a Petri net that extends the type of place (token) to describe different data types. Moreover, arcs

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in CPN are labelled with arc expression functions to describe data operations; transitions in CPN are labelled with guard functions to describe branch conditions. In this way, CPN combines the capabilities of Petri nets and a high-level programming language. Success stories of CPN can be found in many industrial domains, such as network protocols [14], systematic softwares [11, 15], embedded systems [3], e-commerce systems [20], etc.

Explicit-state on-the-fly verification [4, 8, 7] is an universal optimization approach for model checking. It integrates state space generation, product automaton construction and counterexample detection (in LTL (Linear Temporal Logic) model checking, a counterexample is an accepting cycle in product automaton). An advantage of this approach is that the algorithm can give an answer without generating full state space. Though success stories of on-the-fly in P/T nets clearly demonstrate its effectiveness and applicability, there are few works dedicated to directly applying on-the-fly in checking CPN models. As for checking CPN models, the mainstream approach is unfolding [16, 13, 12, 2], which transforms a CPN into an equivalent P/T net and implements model checking on the latter. With unfolding, one can directly apply all successful optimization techniques which are difficult to extend to CPNs on the equivalent net, like Data Decision Diagram (DDD) [5, 1], P-invariants [18]. However, a big disadvantage of unfolding is that the equivalent P/T nets transformed from a CPN tends to be too enormous to be handled, with much more places and transitions. Also, the transformed P/T nets cannot directly describe the system to be verified. If a counterexample is detected by verification process, it is difficult to be directly reflected into the system, which is not friendly for debugging.

Concerned with checking CPN models without unfolding, we present a basic on-the-fly method, named full-info algorithm (FullInfo). It is based on the standard state space generation algorithm [10]. Its core idea is that once a new reachable state m is generated, it calculates a set of all enabled binding elements (we call the set Enbe) in m and stores Enbe together with marking. Enbe serves two purposes. One is to calculate successors of m. Another one is to help check atomic propositions carried by a Büchi automaton state during the generation of product automaton states (or product states for short). For example, some atomic propositions may check enabling of transitions, and enabling of transitions can be reflected by ENBE (for a transition t, if there exists an enabled binding element of t in ENBE, t is enabled. Otherwise, t is not enabled). The algorithm is simple to implement but may generate much redundant information during on-the-fly. A great characteristic of on-the-fly is that it terminates exploration upon a counterexample is detected. Thus, in most cases, the set Enbe of every state m is not fully utilized, cause many successors of m have not been calculated before termination. Also, for some special LTL formulas whose atomic propositions are all related to numbers of tokens (this kind of LTL formulas are called LTLCardinality formulas), Enbe can help

¹ This terminology originated from MCC (Model Checking Contest) which is an annual competition for model checking. https://mcc.lip6.fr/.

nothing, because checking these atomic propositions never refers to information in Enbe. This leads to waste of computing resources and low efficiency.

Besides FullInfo, we introduce two more efficient state space generation methods integrated into on-the-fly, namely, minimum representative algorithm (MinRep) and dynamic exploration algorithm (DynExp). MinRep is inspired by canonical representative in [17]. Its core idea is that for every enabled transition t in a newly calculated reachable state m, only a representative of enabled binding element of t is initially calculated. While in DynExp, none enabled binding element is initially calculated in every newly generated reachable state m. Every enabled binding element in m is calculated on demand when a new successor of m needs generating to start a new path. However, without complete enabled transitions, DynExp is hard to check atomic propositions related to enabling of transitions. Thus, it is limited to LTLCardinality formulas.

In short, the main contributions of this paper are summarized as follows:

- 1. Concerned with LTL model checking of CPN without unfolding, we present an efficient on-the-fly verification method, named *MinRep*. It is an overall improvement of *FullInfo*.
- 2. For LTLCardinality formulas, we present another more efficient on-the-fly verification method, named *DynExp*.
- 3. We implemented *FullInfo*, *MinRep* and *DynExp* and did a number of experiments to demonstrate high efficiency of the latter two algorithms.

The rest of this paper is organized as follows: In Section 2, we introduce the definition of Colored Petri Nets and Linear Temporal Logics. In Section 3, we briefly introduce standard state space generation and on-the-fly verification. Then in Section 4, we specify a binding elements calculation problem from the core part of state space generation and on-the-fly. In Sections 5, 6, 7, we elaborate on FullInfo, MinRep and DynExp. Their strengths and weaknesses are discussed as well. Implementation and experimental results are given in Section 8. Finally, in Section 9, we present our conclusion.

2 PRELIMINARIES

2.1 Colored Petri Nets

In this section, definitions of multi-set and non-hierarchical CPN are cited [10] and definitions of LTL are cited [6, 21]. As a matter of convenience, $Bool = \{false, true\}$ is the set of Boolean types, where true and false are two predicates respectively. Type[v] is the data type of variable v. Type[ex] is the type of expression ex. $EXPR_V$ is an expression constituted by elements from set V.

Definition 1 (Multi-set). Let $S = \{s_1, s_2, s_3, \dots\}$ be a non-empty set. A multi-set m is a function over $S: S \to \mathbb{N}$ that maps each element $s \in S$ into a non-negative

integer $m(s) \in \mathbb{N}$ called the number of appearances (coefficient) of s in m. A multiset m can also be written as a sum (the operator '++' is a natural addition '+' when two elements s_1, s_2 are the same data type, otherwise '++' is just a junction symbol without real meaning):

$$^{++} \sum_{s \in S} m(s)'s = m(s_1)'s_1 + m(s_2)'s_2 + m(s_3)'s_3 + \dots$$

Operators: addition (++), scalar multiplication (**), comparison (\ll =), size (|m|) and subtraction (--) are defined as follows:

- addition: $\forall s \in S, (m_1 + + m_2)(s) = m_1(s) + m_2(s),$
- scalar multiplication: $\forall s \in S, (n**m)(s) = n*m(s),$
- comparison: $m_1 \ll = m_2 \Leftrightarrow \forall s \in S, m_1(s) \leq m_2(s),$
- size: $|m| = \sum_{s \in S} m(s)$,
- when $m_1 \ll = m_2$, subtraction is defined as: $\forall s \in S$, $(m_2 - m_1)(s) = m_2(s) m_1(s)$.

Definition 2 (Non-hierarchical CPN). A non-hierarchical CPN is a nine-tuple $N=(P,T,A,\Sigma,V,C,G,E,I)$, where P,T,A are finite sets of places, transitions and arcs such that $P\cap T=\emptyset$, $A\subseteq P\times T\cup T\times P$, Σ is finite set of non-empty color sets, V is a finite set of typed variables such that $Type[v]\in\Sigma$ for all variables $v\in V,C:P\to\Sigma$ is a color set function that assigns a color set to each place, $G:T\to EXPR_V$ is a guard function that assigns a guard to each transition t such that $Type[G(t)]=Bool,E:A\to EXPR_V$ is an arc expression function that assigns an arc expression to each arc t such that t such th

Definition 3 (Enabling and firing rules). Let $N = (P, T, A, \Sigma, V, C, G, E, I)$ be a non-hierarchical CPN. A marking of N is a function M that maps each place $p \in P$ into a multi-set of tokens $M(p) \in C(p)_{MS}$. A binding of a transition t is a function t that maps each variable t into a value t into a value t into a value t into a transition t is denoted t into a value t into a value t into a pair t into a pair t into a pair t into a pair t into a value t into a value t into a pair t into a pa

- 1. $G(t)\langle b\rangle = true;$
- 2. $\forall p \in P, E(p,t)\langle b \rangle \ll = M(p)$.

When (t,b) is enabled in M, it may occur and is leading to a marking M' (written $M \xrightarrow{(t,b)} M'$), such that $\forall p \in P, M'(p) = (M(p) - -E(p,t)\langle b \rangle) + +E(t,p)\langle b \rangle$. A transition t is enabled in a marking M if and only if $\exists (t,b) \in BE(t)$, (t,b) is enabled in M.

Definition 4 (State space). For a marking M and a marking M', if there exists an enabled binding element (t,b) such that $M \xrightarrow{(t,b)} M'$, M' is said to be immediately reachable from M; if there exists an sequence of binding element $(t_1,b_1)(t_2,b_2)\dots(t_n,b_n)$ such that $M \xrightarrow{(t_1,b_1)} M_1 \xrightarrow{(t_2,b_2)} M_2 \dots \xrightarrow{(t_n,b_n)} M'$, M' is said to be reachable from M, written $M \xrightarrow{*} M'$. The state space of a CPN consists of the set $R(m_0) = \{m \mid m_0 \xrightarrow{*} m\}$ of states reachable from the initial state. Each state $m \in R(m_0)$ is called a reachable state².

2.2 Linear Temporal Logics

Linear Temporal Logic (abbreviated as LTL) is used to describe properties of a system execution. It consists of a non-empty finite set of atomic propositions AP, Boolean operators \neg (negation), \lor (disjunction) and \land (conjunction), and temporal operators X (next), U (until), R (release), F (eventually) and G (always). In LTL model checking, the negation of a formula will be transformed into a Büchi automaton. There are many approaches to construct a Büchi automaton from the LTL formula [6, 19].

Definition 5 (Syntax of LTL). The syntax of LTL is defined as follows:

$$\phi ::= p \mid \neg \varphi \mid \varphi \vee \psi \mid \varphi \wedge \psi \mid X\varphi \mid \varphi U\psi \mid \varphi R\psi \mid F\varphi \mid G\varphi$$

where p is an atomic proposition and ϕ , φ , ψ are well-formed LTL formulas. Referring to MCC and Wolf's [21] provisions for atomic propositions, we make the following provisions for an atomic proposition p:

$$p ::= TRUE \mid FALSE \mid FIREABLE(t) \ (t \in T) \mid DEADLOCK$$
$$\mid k_1 p_1 + \dots + k_n p_n \le k \ (k_i, k \in \mathbb{Z}, p_i \in P)$$

Let state m be the current state, FIREABLE(t) holds if only if t is enabled in m, DEADLOCK holds if and only if there are no transitions are enabled in m, $k_1p_1 + \cdots + k_np_n \le k$ holds if and only if $k_1M(p_1) + \cdots + k_nM(p_n) \le k$ in m.

State is a snapshot of a system, marking is a distribution of tokens. Though a state m can be uniquely identified by a marking M, they are different concepts. Throughout the paper, we use lower case m (subscripts or superscripts will be used if necessary) to represent a state, upper case M(subscripts or superscripts will be used if necessary) to represent the marking of m.

Definition 6 (Semantics of LTL). Let AP be a non-empty finite set of atomic propositions, $\xi = x_0 x_1 x_2 \dots$ be a sequence over alphabet 2^{AP} , ϕ , φ , ψ be LTL formulas. We write ξ_i for the suffix of ξ starting at x_i . The semantics $\xi \models \phi$ (ξ models ϕ) is defined as follows:

- $\xi \models p$, iff $p \in x_0$ for $p \in AP$,
- $\xi \models \neg \phi$, iff $\xi \not\models \phi$,
- $\xi \models \varphi \lor \psi$, iff $\xi \models \varphi$ or $\xi \models \psi$,
- $\xi \models X\phi$, iff $\xi_1 \models \phi$,
- $\xi \models \varphi U \psi$, iff $\exists i \geq 0, \ \xi_i \models \psi \land (\forall j < i, \xi_i \models \varphi)$.

Other operators (\land, R, F, G) can be derived from the above operators (X, U, \neg) : $\varphi \land \psi \equiv \neg(\neg \varphi \lor \neg \psi)$; $\varphi R \psi \equiv \neg(\neg \varphi U \neg \psi)$; $F \phi \equiv (TRUE)U \phi$; $G \phi \equiv \neg(F \neg \phi)$.

3 STANDARD STATE SPACE GENERATION AND ON-THE-FLY

3.1 Standard State Space Generation

The standard state space generation [10] works on three sets: Node, Unprocessed, Edges. Node stores reachable states. Unprocessed consists of states whose successors have not yet been calculated. Edges stores arcs. As illustrated in Algorithm 1, the algorithm firstly initializes Node, Unprocessed with initial state m_0 and Edges with empty set. Then it selects a reachable state m in Unprocessed and calculates all enabled binding elements in m. Each enabled binding element that occurs will lead to a reachable state m' and an arc from m to m'. If m' has not yet been encountered, it will be added into Node and Unprocessed. The algorithm terminates with full state space.

3.2 On-the-Fly

On-the-fly method was first proposed in [4]. The main idea is integrating state space generation, product automaton construction and detecting counterexamples (in LTL model checking, a counterexample is an accepting cycle in product automaton). In more detail, for a given product state p:(m,b) (a product state is composed by a reachable state m and an automaton state b), it calculates a successor m' of m, and a successor b' of b. if all atomic propositions carried by b' are satisfied in m', then a product state p':(m',b') is generated. If some conditions are triggered, on-the-fly will implement counterexample detection, i.e., if on-the-fly finds the successor p' of p is an encountered product state where it may form a cycle, then on-the-fly will check that. This idea can be illustrated by Algorithm 2. Line 1 is state space generation, lines 3–4 are the product state generation and the line 6 is counterexample detection. As for counterexample detection, there are several ways to do that.

Algorithm 1 Standard state space generation

```
1: Nodes \leftarrow \{m_0\}
2: Unprocessed \leftarrow \{m_0\}
3: Edges \leftarrow \emptyset
 4: while Unprocessed \neq \emptyset do
        Select a Marking m in Unprocessed
        Unprocessed \leftarrow Unprocessed -\{m\}
6:
        for all binding elements (t, b) such that (t, b) is enabled in m do
 7:
            Calculate m' such that m \xrightarrow{(t,b)} m'
8.
            Edges \leftarrow Edges \cup \{(m, (t, b), m')\}
9.
            if m' \notin \text{Nodes then}
10:
                Nodes \leftarrow Nodes \cup \{m'\}
11:
                Unprocessed \leftarrow Unprocessed \cup {m'}
12:
13:
14.
        end for
15: end while
```

Like nested depth-first search algorithm [4], TCHECK³ algorithm [7] and DCHECK algorithm [7].

```
Algorithm 2 on-the-fly
```

```
Input: p:(m,b): a product state
Output: true or false: checking result
 1: for m' \leftarrow \text{NEXTSUCCESSOR}(m) \neq \text{'no more'} do
       for all b' \in SUCCESSOR(b) do
 2:
           if m' satisfies all atomic propositions carried by b' then
 3:
               Generate a produce state p' :: (m', b')
 4.
               if p' has been encountered then
 5:
                   Accepting cycle detection
 6:
                   if \exists an accepting cycle then
 7:
                      Terminate with false
 8:
                  end if
 9.
               else
10:
                  on-the-fly(p')
11:
               end if
12:
13:
           end if
       end for
14:
15: end for
```

 $^{^3}$ The main procedure of TCHECK and DCHECK are non-recursive functions, and they work much more efficiently than nested depth-first search. FullInfo, MinRep and DynExp are integrated into TCHECK algorithm. More details can be referred to [7].

4 ENABLED BINDING ELEMENTS CALCULATION PROBLEM

Enabled binding elements are vitaly important during state space generation. Firstly, all successors of a reachable state are controlled by enabled binding elements (lines 7–8 in Algorithm 1 and line 1 in Algorithm 2). Secondly, enabled binding elements plays a part in product state generation (line 3 in Algorithm 2), because some atomic propositions may check enabling of some transitions, i.e., FIREABLE(t) atomic propositions. The core problem that we encounter is: given a reachable state m, in which way to explore binding element space BE to find enabled binding elements in m to calculate successors of m and generate product states. Different solutions to this problem lead to huge different performances. Intuitively, we may come up with that upon a new reachable state m is generated, explore BE exhaustively at once to get all enabled binding elements ENBE in m and store ENBE in case to use. In this way, every time on-the-fly backtracks to m, the process can easily fetch a next enabled binding element from ENBE to calculate another successor of m. This is exactly how FullInfo works. We will detail it in the next section.

5 FULLINFO

The core idea of FullInfo is very simple: upon getting a new reachable state m, it calculates a set of all enabled binding elements in m called Enbe and stores Enbe together with marking M immediately. Then it uses Enbe to generate different successors of m and product states. The technical difficulties lie in how to get all enabled binding elements and how to manage them.

5.1 How to Get All Enabled Binding Elements

Traversing t's binding space B(t) is essentially a combination problem that assigns a value $b(v) \in Type[v]$ to each variable $v \in Var(t)$. The binding space B(t) can be depicted as a tree (we name it t's binding space tree). Assume that $t \in T$ is a transition, |Var(t)| = k such that $Var(t) = \{v_1, v_2, \dots, v_k\}$ and for each variable v_i , $|Type[v_i]| = n_i$ such that $Type[v_i] = \{c_{i1}, c_{i2}, \ldots, c_{in_i}\}$, then B(t) can be depicted as a tree in Figure 1. The depth of this tree is equal to the number of variables in Var(t). All direct successors of a node are overall mapping cases of next variable. For example, the direct successors of node $v_1 = c_{11}$ list complete mapping cases of next variable v_2 , which is from $v_2 = c_{21}$ to $v_2 = c_{2n_2}$ (we use horizontal ellipsis to represent all omitted nodes in its layer and vertical ellipsis to represent all omitted child nodes of one node). A path from root node to a leaf node is a specific binding of t, and all paths like this constitute t's binding space B(t). We use a recursive function to traverse this tree to get all enabled binding elements. The function is presented in Algorithm 3. When the depth is lower than |Var(t)|, the function tries to assign a color to the variable which corresponds to the depth and then recurves down. If the depth is equal to or greater than |Var(t)| which means the function

reaches a leaf node and a complete binding b is obtained, then it begins to check the enabling of (t,b). The specific checking procedure lies in lines 2–9. If (t,b) is enabled, it will be added into ENBE. After implementing this function on each transition $t \in T$, the complete ENBE will be obtained.

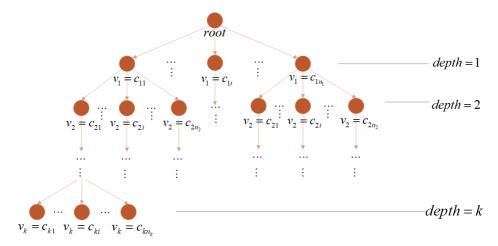


Figure 1. Binding space tree

Algorithm 3 getENBE(m, t, b, depth)

```
Input: m: reachable state, t: Transition, b: Binding, depth: int
Output: Enbe: a set stores enabled binding elements
 1: if depth \ge |Var(t)| then
        if \neg G(t)\langle b \rangle then
 2:
            return
 3:
        end if
 4:
        for all p \in {}^{\bullet}t do
 5:
            if \neg (E(p,t)\langle b\rangle \leq M(p)) then
                                                                           \triangleright M is m's marking
 6:
                return
 7:
            end if
 8:
        end for
 9:
        Enbe.ADD(b)
10:
11: else
        for all c \in Type[v_{depth}] do
12:
            b[depth] \leftarrow c
13:
            getENBE(m, t, b, depth + 1)
14:
        end for
15:
16: end if
```

5.2 How to Manage All Enabled Binding Elements

In this subsection, we focus on how to take advantage of ENBE to serve for successor reachable states generation and product states generation. We use a two-level queue as data structure for ENBE. The first level queue stores enabled transitions, each enabled transition has a second level queue consisting of its bindings which render it enabled. Figure 2 is an example of

ENBE =
$$\{(t_1, b_{11}), (t_1, b_{12}), (t_2, b_{21}), (t_2, b_{22}), (t_2, b_{23}), (t_3, b_{31})\}.$$

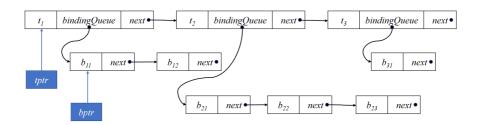


Figure 2. Data structure for Enbe

In the data structure of ENBE, there are two pointers, tptr and bptr, respectively pointing to an enabled transition and a binding of it. They are used to represent an enabled binding element, i.e., in Figure 2 they represent (t_1, b_{11}) . Each time an enabled binding element occurs, bptr will move to next binding of the current queue. If next binding does not exist, i.e., it reaches the tail of the queue, tptr will move to next transition and bptr will point to the head of its bindingQueue. By this, the process can obtain different successors of a reachable state and this procedure is one possible way how line 7 in Algorithm 1 and line 1 in Algorithm 2 work.

Another crucial role of ENBE is to help generate a product state. For example, let $F\alpha$ be a LTL formula, where α is an atomic proposition $FIREABLE(t_2)$. During checking process, every state needs to check if t_2 is enabled in it. To do this, every state just needs to check its ENBE. If t_2 appears in the first level queue, it is enabled, otherwise it is not.

With FullInfo, we can basically solve the enabled binding elements calculation problem. The two core parts, successor reachable states generation and product states generation, can be done easily with the aid of Enbe. But a conspicuous disadvantage is that it may generate much redundant information. Or in other words, many states' Enbe may not be fully utilized. For example, if on-the-fly reports a checking result without generating the whole state space, that means there must exist some states where some enabled binding elements have not yet occurred and these enabled binding elements remain to be redundant. Another case is when none of atomic propositions of the LTL formula is form of FIREABLE(t) or DEADLOCK, Enbe can do nothing to help in the product state generation. This disadvantage is

particularly obvious when a CPN's binding element space is huge or when on-the-fly detects a counterexample along a path with few backtrackings. Here, Figure 3 is an example to demonstrate the second case. Figure 3 is a partial state space of a CPN. We use solid cycles to represent reachable states, arrows marked by transitions to represent enabled transitions in a reachable state, solid squares to represent enabled binding elements which have occurred and hollow squares to represent enabled binding elements which have not yet occurred. If on-the-fly detected a counterexample $S_0 \rightarrow S_1 \rightarrow S_2 \rightarrow S_0$ after generating S_0 , S_1 , S_2 and then terminates, then the computing resources allocated for calculating the hollow squares are wasted because they had never been used during checking process. If on-the-fly went deeper along this path and detected a counterexample, the waste would be worse. Therefore, we need another algorithm to solve the enabled binding elements calculation problem.

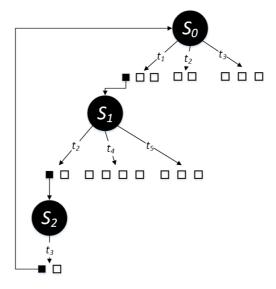


Figure 3. Partial state space

6 MINREP

In this section, we develop another solution to the enabled binding elements calculation problem. According to the definition of enabling of transitions (in Definition 3), if there exists one enabled binding element of a transition t, t is proven to be enabled. Thus, as for checking atomic propositions during product state generation, it is unnecessary to calculate complete ENBE in each reachable state. The core idea of MinRep is to specify an order over B(t) such that $(B(t), \prec)$ for each transition t. And for t's binding element space, this algorithm only initially calculates one en-

abled representative which is the smallest enabled one in BE(t). Certainly, if t is not enabled, there will not be such a representative. This idea is inspired by canonical representative [17].

Before presenting the order $(B(t), \prec)$, we firstly specify an order (C, \prec) over each color set $C \in \Sigma$. Here are the orders:

- 1. (C, \prec) : $\forall c_i, c_j \in C$, $c_i \prec c_j$ iff i < j. Here the index i, j can be arbitrarily defined. Typecially we use the index in data structure storing color set C, i.e., sequence table.
- 2. $(B(t), \prec)$: $Var(t) = \{v_1, v_2, \ldots, v_n\}$, $\forall b_i, b_j \in B(t)$, $b_i = \langle c_{i1}, c_{i2}, \ldots, c_{in} \rangle$, $b_j = \langle c_{j1}, c_{j2}, \ldots, c_{jn} \rangle$, $b_i \prec b_j$ iff $\exists k, 1 \leq k \leq n, c_{ik} \prec c_{jk}$ and $\forall m, 1 \leq m < k, c_{im} = c_{jm}$. $(B(t), \prec)$ can be regarded as a lexicographical order induced by the vector of binding.

Calculating representative is similar to Algorithm 3. The minor difference is that for *MinRep*, upon getting an enabled binding element, it terminates. More specifically, we just need to insert a terminate clause after line 10. All representatives are organized in a set, and we name it Ent. Here we use a queue to organize Ent. Figure 4 is an example of

ENT =
$$\{(t_1, b_{11}), (t_2, b_{21}), (t_3, b_{31})\}.$$



Figure 4. Data structure for Ent

In this data structure, tptr is a pointer pointing to a transition occurring last time and nextb is a binding that is prepared to calculate a successor reachable state next time (initially it equals to the binding part of the related representative). When on-the-fly backtracks to a reachable state m, the program uses nextb to calculate a successor m' of m and tries to check bindings behind (tptr->nextb) to update nextb. If there are no more bindings related to tptr, tptr will move to next representative, which indicates the binding element space of prior transition has been explored exhaustively. The successor generation procedure is presented in Algorithm 4 where (m.tptr->t) is the transition of corresponding representative which pointed by tptr. Function UPDATE(m) is to update (tptr->nextb). It regards the binding vector as a n-digit number, where n equals to |Var(m.tptr->t)|, and tries to implement increment '++' on this number. During the increment operation, it may trigger carries (lines 7–8) like numbers. Function NEXTSUCCESSOR(m) is to get another successor of m according to binding element (m.tptr->t, m.tptr->nextb).

As for checking atomic propositions, ENT together with marking is right enough. As for FIREABLE(t) propositions, MinRep checks ENT of current reachable state m. If there is a related representative of t, t is enabled, otherwise, it is not. As for DEADLOCK propositions, MinRep also checks ENT of current reachable state m. If ENT is empty, the propositions are satisfied, otherwise, it is not. As for $k_1p_1 + \cdots + k_np_n \leq k$ propositions, MinRep checks them by the marking M of current reachable m. if $k_1M(p_1) + \cdots + k_nM(p_n) \leq k$ holds, the propositions are satisfied, otherwise, it is not.

Algorithm 4 MinRep

```
Input: m: current reachable state
Output: m': successor of m or 'no more'
 1: function UPDATE(m)
        while m.tptr \neq NULL do
            (t',b') \leftarrow (m.tptr->t,m.tptr->nextb)
 3:
                                                                  \triangleright Var(t') = \{v_1, v_2, \dots, v_n\}
            n: int \leftarrow |Var(t')|
 4:
            for i from n to 1 do
 5:
                c: \operatorname{color} \leftarrow \operatorname{NEXTCOLOR}(b[i])
 6:
                if c = \text{`no more'} then
 7:
                    b'[i] \leftarrow \text{first color of } Type[v_i]
 8:
                    continue
 9:
                else
10:
                    b'[i] \leftarrow c
11:
                    if (t',b') is enabled in m then
12:
                        m.tptr - > nextb \leftarrow b'
13:
                        return
14:
                    end if
15:
                end if
16:
            end for
17:
            tptr \leftarrow tptr - > next
18:
        end while
19:
        return
20:
21: end function
22:
23: function NEXTSUCCESSOR(m)
        if tptr = NULL then
24:
            return 'no more'
25:
        else
26:
            Calculate m' such that m \xrightarrow{(m.tptr->t,m.tptr->nextb)} m'
27:
            UPDATE(m)
28:
            return m'
29:
        end if
31: end function
```

For each reachable state m, MinRep only calculates partial information from m's binding element space which is just enough to handle all kinds of atomic propositions. Compared with FullInfo, redundant information is much less and efficiency would be higher. But it has the same disadvantage that if all atomic propositions of the LTL formula are neither form of DEADLOCK nor FIREABLE(t), i.e., LTLCardinality formulas, ENT can help nothing and remain to be redundant. So we need another more efficient algorithm to handle LTLCardinality formulas.

7 DYNEXP

In this section, we develop another algorithm dedicated to handling LTLCardinality formulas. As for checking atomic propositions of this formula type, it is unnecessary to calculate any enabled binding elements. The sole function of enabled binding elements here is to calculate successors. In order to be more efficient, DynExp will not initially calculate any enabled binding element in each newly calculated reachable state m. Instead of calculating all enabled binding elements at once, enabled binding elements are obtained dynamically. Once an enabled binding element is obtained, the algorithm will let it occur immediately and calculate a successor m' of m, then continue on-the-fly on m'. To obtain different successors of a given reachable state m when on-the-fly backtracks to m, DynExp extends the orders defined in Section 6 to the whole binding element space BE such that (BE, \prec) , and each reachable state would record the binding element that occurred last time, called lastbe. In this way, when on-the-fly backtracks to m, it can check binding elements behind lastbe until an enabled one is detected or there are no more in BE.

Before presenting the order (BE, \prec) , we firstly specify an order (T, \prec) over transition set T. They are defined as follows:

- 1. (T, \prec) : $\forall t_i, t_j \in T$, $t_i \prec t_j$ iff i < j. Here the index value i, j can be arbitrarily defined. Typically we use its index value in a specific data structure that stores the transition set.
- 2. (BE, \prec) : $\forall (t_i, b_{ik}), (t_j, b_{jm}) \in BE, (t_i, b_{ik}) \prec (t_j, b_{jm}) \text{ iff } t_i \prec t_j \text{ or } i = j, b_{ik} \prec b_{jm}.$

With the aid of order (BE, \prec) and lastbe, the algorithm can iterate over BE exhaustively to get different enabled binding elements in each reachable state. We specify three functions to implement the idea, namely, NEXTBINDING((t,b)), NEXTTRANSITION((t,b)) and NEXTSUCCESSOR(m). They are illustrated in Algorithm 5. Function NEXTBINDING((t,b)) is similar to function UPDATE(m) illustrated in Algorithm 4. It is to fetch (t,b)'s next binding of transition t according to order (BE, \prec) . Where function NEXTCOLOR(c) is to get color c's next color in c's color set C by order (C, \prec) . Function NEXTTRANSITION((t,b)) is very simple. Its job is to fetch next transition of t according to order (T, \prec) , and initiate the binding (lines 23–25). If there are no more transitions, 'NULL' will be returned.

Algorithm 5 DynExp

```
Input: m: current reachable state
Output: m': successor of m
 1: function NEXTBINDING((t, b))
        (t',b') \leftarrow (t,b)
        n: int \leftarrow |Var(t)|
 3:
        for i from n to 1 do
 4:
            c: color \leftarrow NEXTCOLOR(b[i])
 5:
            if c = \text{`no more'} then
 6.
                b'[i] \leftarrow \text{first color of } Type[v_i]
                continue
            else
 9:
                b'[i] \leftarrow c
10:
                return (t', b')
11:
            end if
12:
        end for
13:
        return 'no more'
15: end function
16:
17: function NEXTTRANSITION((t, b))
18:
        (t',b'): binding element
        t' \leftarrow t.index++
19:
        if t' = \text{`no more'} then
20:
            return 'no more'
21:
        else
22:
23:
            for i from 1 to |Var(t')| do
                b'[i] \leftarrow \text{first color of } Type[v_i]
24:
            end for
25:
        end if
26:
        return 'no more'
27:
28: end function
29:
30: function NEXTSUCCESSOR(m)
        repeat
31:
            repeat
32:
                (t,b) \leftarrow \text{NEXTBINDING}(m.lastbe)
33:
                if (t, b) is enabled in m then
34:
                   Calculate m' such that m \xrightarrow{(t,b)} m'
35:
                   return m'
36:
                end if
37:
            until (t,b) = 'no more'
38:
39:
            (t,b) \leftarrow \text{NEXTTRANSITION}(m.lastbe)
        until (t, b) = 'no more'
40:
        return 'no more'
42: end function
```

Function NEXTSUCCESSOR(m) keeps calling NEXTBINDING(m.lastbe) to iterate over B(t), trying to find an enabled one. If there are no more or do not exist at all, it will move to next transition by calling NEXTTRANSITION(m.lastbe). Upon obtaining an enabled binding element (t,b), (t,b) will occur immediately leading to a successor m' of m and terminates this function. Or if there are no more enabled binding element, NEXTSUCCESSOR(m) will return 'no more'.

As for generating product states, we have nothing to worry about, because checking LTLCardinality formulas just need information of markings and markings are never absent.

Because upon getting a successor, the algorithm terminates exploring binding element space and continues on-the-fly, any enabled binding element and corresponding successor are calculated on demand during the checking process. Hence, no redundant information is generated and *DynExp* would be more efficient than *FullInfo* and *MinRep*. However, it is limited to LTLCardinality formulas. It sacrifices applicability for greater efficiency.

8 EXPERIMENT

We implemented all three algorithms in C++, and they are all integrated into the non-recursive on-the-fly TCHECK. The source code is available from:

- FullInfo: https://github.com/Tj-Cong/EnPAC_CPN,
- *MinRep*: https://github.com/Tj-Cong/EnPAC_CPN_F,
- *DynExp*: https://github.com/Tj-Cong/EnPAC_CPN_C.

We get testing data from MCC. There are two kinds of models provided by MCC:

- Academic models: these were designed in universities by researcher, to benchmark some tools, to illustrate a typical situation or within the context of academic projects and cooperations.
- Industrial models: these where designed within the context of industrial projects.

Both kinds of models have practical meanings and each model is provided with a file describing it which can be found from https://mcc.lip6.fr/models.php. Each model can result in several instances due to the scaling parameter (the parameters are often indicated at the end of its instance name). There are two kinds of LTL formulas. One is called LTLCardinality formulas whose atomic propositions are all form of $k_1p_1 + \cdots + k_np_n \leq k$. Another kind is called LTLFireability formulas whose atomic propositions are all form of FIREABLE(t). Based on this testing data, we have done two sets of experiments. One is designed to measure the performance on LTLcardinality formulas. Another one is designed to measure the performance on LTLfireability formulas. They are both implemented on a Linux PC with Intel(R) Core(TM) i7-7700HQ CPU@2.80 GHz and 16 GB RAM. Operating system is Ubuntu 18.04 LTS.

As for the first experiment, all three algorithms were tested on four different instances with different size of binding element space. Each instance is checked by two formulas. Testing time for each formula is limited to 300 seconds, and if one algorithm does not finish checking one formula within 300 seconds, the corresponding table entry will be marked as "?". Memory for each formula is limited to 16 GB and if one algorithm cannot finish checking one formula within 16 GB, the corresponding table entry will be marked as "Overflow". Of course, the three algorithms are set to traverse paths in the same order. The result is presented in Table 1. |States| represents the number of states explored by on-the-fly before termination. The unit of time is seconds; the unit of memory is MB; the size of binding element space |BE| is calculated by:

$$|BE| = \sum_{t \in T} \left(\prod_{v \in Var(t)} (|Type[v]|) \right).$$

From Table 1, we can find that DynExp always consumes the least time and memory. Thus, we can conclude that DynExp is the most efficient algorithm for LTLcardinality formulas, no matter with respect to memory consumption or time used. MinRep ranks the second and FullInfo is the least efficient. Beginning from Formula 2 of DWM-COL-40, memory for FullInfo overflows. And beginning from Formula 1 of GRA-COL-11, time for MinRep runs out of 300 seconds. When |BE| goes larger, the advantage of dynamic exploration becomes more salient. By checking Formula 1 and Formula 2 of every instance, we can also find out that the more states on-the-fly explores, more obvious the advantage of DynExp is.

26.11	1.55	O.T. 1.01		*** ·**		OT 112	57576	×0.7 + 0.1
Models	ALD-COL-10 ¹		DWM-COL-40 ²		GRA-COL-11 ³		DVM-COL-16 ⁴	
Places	20		11		5		6	
Transitions	15		8		7		7	
BE	132		12 800		2 705 087		4 433 952	
formulas	1	2	1	2	1	2	1	2
States	38 115	43 109	20 936	1251201	286755	545 605	178433	457 369
Time (FullInfo)	2.510	3.193	36.417	?	?	?	?	?
Time (DynExp)	0.899	1.644	8.089	28.381	27.462	80.120	10.377	57.256
Time (MinRep)	0.905	2.751	14.4249	151.37	> 300	> 300	> 300	> 300
Memory (FullInfo)	506.367	529.363	2766.367	Overflow	Overflow	Overflow	Overflow	Overflow
Memory (DynExp)	433.348	449.348	504.344	8 080.348	871.244	1387.344	1181.348	2,518.352
Memory (MinRep)	433.348	449.348	508.348	8 252.348	?	?	?	?

¹The full name is AirplaneLD-COL-0010.

Table 1. Comparison on LTLCardinality formulas

As for the second experiment, FullInfo and MinRep were implemented on four different instances with different size of binding element space. Same as the first experiment, each instance is checked by two formulas and each formula is limited to 300 seconds and 16 GB. The result is presented in Table 2. Obviously, MinRep works more efficiently than FullInfo. Comparing Formula 2 of FR-COL-G005 with

²The full name is DatabaseWithMutex-COL-40.

 $^{^3{\}rm The}$ full name is Global ResAllocation-COL-11.

 $^{^4}$ The full name is DrinkVendingMachine-COL-16.

Formula 1 of GRA-COL-9, we can find that for MinRep, the memory consumption is much lower in GRA-COL-09, while for FullInfo, the memory consumption is much higher in GRA-COL-9, because the size of binding element space is much bigger. We can also conclude that when |BE| goes larger, the advantage of MinRep becomes more obvious. Also, the more states on-the-fly explores, more salient the advantage of MinRep is.

Models	ALD-COL-50 ¹		DWM-COL-40 ²		FR-COL-G005 ³		GRA-COL-9 ⁴		
Places	20		11		104		5		
Transitions	15		8		66		7		
BE	63	612		12 800		134 480		1003437	
formulas	1	2	1	2	1	2	1	2	
States	7	209	2077	9 596	115 121	31 812	36424	?	
Time (FullInfo)	0.104	0.121	3.054	13.600	22.739	57.739	?	?	
Time (MinRep)	0.091	0.105	1.628	5.092	3.609	7.097	113.148	>300	
Memory (FullInfo)	327.359	334.357	630.359	2222.359	1 791.363	3 894.363	Overflow	Overflow	
Memory (MinRep)	327.351	327.351	347.355	405.355	712.351	1 553.348	400.348	?	

¹The full name is AirplaneLD-COL-0050.

Table 2. Comparison on LTLFireability formulas

From the two experiments, we can conclude that no matter what kind of LTL formulas is, *MinRep* is always more efficient that *FullInfo*. While for LTLCardinality formulas, *DynExp* works best.

9 CONCLUSIONS

We have presented a basic state exploration method and two more efficient ones under the framework of on-the-fly. The basic one, FullInfo, simply calculates all enabled binding elements for every newly generated reachable state. It is easy to implement but efficiency is low. MinRep is 'semi-dynamic'. It calculates all enabled transitions for every newly generated reachable state, but for each enabled transition, only a minimum representative of enabled binding elements is initially calculated, others are calculated dynamically on demand. While DynExp is 'fully-dynamic'. Every enabled binding element is calculated on demand by on-the-fly. As for applicability, FullInfo=MinRep>DynExp. As for efficiency, DynExp>MinRep>FullInfo.

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 $^{^2{\}rm The}$ full name is DatabaseWithMutex-COL-40.

 $^{^3{\}rm The}$ full name is Family Reunion-COL-L00200M0020C010P010G005.

⁴The full name is GlobalResAllocation-COL-09.

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