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CORRELATION COEFFICIENT MEASURE OF INTUITIONISTIC FUZZY GRAPHS WITH APPLICATION IN MONEY INVESTING SCHEMES

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Abstract. Intuitionistic fuzzy graphs are extensions of fuzzy graphs that preserve the dualism characteristics of fuzzy graphs and have a stronger capacity to describe ambiguity in actual decision-making issues than fuzzy graphs. In this research paper, the Laplacian energy and correlation coefficient of intuitionistic fuzzy graphs are computed for finding group decision-making problems that are supported by intuitionistic fuzzy preference relations. We propose a novel method for calculating establishments' comparative position loads by manipulating the undecided corroboration of IFPR and the correlation coefficient of one personality IFPR to the other items. As a result, we comprehend a large number of establishments in the detailed IFPR and devise a correlation coefficient process to investigate the significance of alternatives and the best of the alternatives. Finally, we present a collaborative decision-making technique in a money-investing scheme, and that idea may be devised in disparate beneficial investing schemes.

Keywords: Intuitionistic fuzzy preference relation, intuitionistic fuzzy adjacency matrix, intuitionistic fuzzy laplacian matrix, intuitionistic fuzzy graph, Laplacian energy, correlation coefficient, group decision-making problem

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1 INTRODUCTION

FS	Fuzzy sets
FG	Fuzzy graph
IFS	Intuitionistic fuzzy set
IFG	Intuitionistic fuzzy graph
IFPR	Intuitionistic fuzzy preference relation
IFAM	Intuitionistic fuzzy adjacency matrix
IFLM	Intuitionistic fuzzy Laplacian matrix
LE	Laplacian energy
CC	Correlation coefficient
GDMP	Group decision making problem
FMF	Fuzzy membership function
FNMF	Fuzzy non-membership function
MVs	Membership values
NMVs	Non-membership values

Table 1. Nomenclature

Zadeh [1] proposed the notion of fuzzy sets. The range of truth value of the membership relation is the interval $[0, 1]$, which is a property of FS. To address the ambivalence and doubt regarding the membership degree, Atanassov [2] added a new degree, termed as degree of non-membership, to the FS concept in 1986. In a fuzzy set, one excluding the degree of membership functions is known as the indecision degree or non-membership degree of a particular component, and it is thus totally stable. However, in authentic or many instances, there is a degree of ambivalence seen between membership functions, and thus they are independent. Zadeh [3] presented the idea of a fuzzy graph relation, which has been used to analyse cluster patterns. Kaufmann [4], created the concept of FG based on Zadeh's hazy relations. Rosenfeld [5] proposed the notion and construction of the FG. Gutman [6] and Balakrishnan [7] defined graph energy in chemistry, as well as its importance to the total π-electron energy of specific compounds, and identified superior and inferior graph energy limits. In [8] Anjali and Mathew investigated the energy of a FG. The LE of a FG was presented by Sharbaf and Fayazi [9]. The idea of a FG was expanded by Parvathi and Karunambigai [10] to include an IFG. The familiarity with the LE of a FG was applied to the LE of an IFG by Basha and Kartheek in [11]. IFG is one of the most popular and unrivalled extensions of IFS perception. Recently, Falehi [12, 13, 14] has successfully performed IFPRs and their executions using a variety of methodologies. Many novel notions about extended architectures of fuzzy graphs were proposed by Akram et al. [15, 16, 17, 18, 19, 20, 21], and their related implications in decision-making. Also, to choose the optimum alliance partner, Ramesh et al. [22] used a GDM procedure that connected the TOPSIS method with IFG.

In an intuitionistic ambiguous scenario, focusing on the variance and covariance of the IFS, Xuan [23] devised a method for determining the correlation coefficient, the value of which is in $[-1, 1]$. Ye [24] proposed a technique in GDMP based on weighted correlation coefficients using LE is presented for particular situations when the knowledge about criterion weights for alternatives is totally unknown. Also, several statistical methods have been executed by Akula and Sharief Basha [25], Zeng and Li [26], Mitchell [27], Huang ad Guo [28], Szmidt and Kacprzyk [29]. Garg and Rani [30], Khaleie and Fasanghari [31], etc. offered several statistical methods for handling decision-making circumstances by using intuitionistic fuzzy sets to represent the quality of the substitutes and fuzzy values to express the weight of each criterion.

According to intuitionistic fuzzy set research, it is crucial to consider this extension concept. It motivates us to think about IFGs and their applications. In this paper, we provide a strategy for solving GDM issues when the weights (loads) of the criteria are completely unknown and the alternatives are solely determined by the IFG. To address ambiguous information criteria, we use the LE measure to calculate the relative weights based on each decision matrix. To satisfy the total weight vector requirement, we combine each LE weight that was received. The correlation coefficient metric is used to evaluate IFG alternatives, and the best ones are then chosen by calculating the correlation degree for each ranking of the alternatives.

The remainder of this article is structured as follows: The essential principles, covariance, and correlation coefficient measures of IFG are presented in Section 2. Group decision-making is presented in Section 3, utilising IFG's Laplacian energy and correlation coefficient technique. The appropriate application is found in Section 4. Ultimately, the conclusion of the article is presented in Section 5.

2 PRELIMINARIES

Definition 1. An IFG $G_i = (V, E, \mu, \nu)$ is defined as a FG with the nodes set V and the paths set E, where μ is a FMF specified on $V \times V$ and ν is a FNMF, then we specify $\mu(v_i, v_j)$ by μ_{ij} and $\nu(v_i, v_j)$ by ν_{ij} so as that

- $0 \leq \mu_{ij} + \nu_{ij} \leq 1$,
- $0 \leq \mu_{ij}, \nu_{ij}, \pi_{ij} \leq 1$,

where $\pi_{ij} = 1 - (\mu_{ij} + \nu_{ij})$.

Definition 2. An IFAM is well-defined for an IFG $G = (V, E, \mu, \nu)$ by $A(G_i) = [a_{ij}]$, where $a_{ij} = (\mu_{ij}, \nu_{ij})$. It is worth noting that μ_{ij} denotes the strength of the membership bond between v_i and v_j and ν_{ij} denotes the strength of the non-membership bond among both v_i and v_j.

Definition 3. An IFAM can be represented by two matrices, one carrying MVs as well as the other carrying NMVs. So that we represent this matrix as

$$A(G_i) = [(A_\mu(G_i)), (A_\nu(G_i))],$$

where $A_\mu(G_i)$ is the intuitionistic fuzzy membership matrix and $A_\nu(G_i)$ is the intuitionistic fuzzy non-membership matrix.

Definition 4. The Eigen roots of an IFAM are described as (Y, Z), where Y represents the set of latent roots of $A_\mu(G_i)$ and Z represents the set of latent roots of $A_\nu(G_i)$.

Definition 5. Permit $A(G_i)$ as an IFAM and $D(G_i)$ specified by $[d_{ij}]$ as the degree matrix of an IFG. Then IFLM of IFG is defined as

$$L(G_i) = D(G_i) - A(G_i).$$

An IFG's Laplacian matrix can be represented as two matrices, one with MV elements and the other with NMV elements i.e.

$$L(G_i) = [(L(\mu_{ij})), (L(\nu_{ij}))].$$

Definition 6. Consider an IFG $G_i = (V, E, \mu, \nu)$ and λ_i, θ_i are the latent roots of Intuitionistic fuzzy adjacency matrix $A(G_i)$. Then the LE of IFG is described as follows:

$$LE(G_i) = [LE(A_\mu(G_i)), LE(A_\nu(G_i))],$$

where $A_\mu(G_i)$ and $A_\nu(G_i)$ are the membership matrix and non-membership matrix of $A(G_i)$ of an IFG, and λ_i, θ_i are the latent roots of $A_\mu(G_i)$ and $A_\nu(G_i)$. Also, $LE(A_\mu(G_i))$ and $LE(A_\nu(G_i))$ gives the Laplacian energies of membership matrix $A_\mu(G_i)$ and non-membership matrix $A_\nu(G_i)$ of IFG. The LE of $(A_\mu(G_i))$ and $(A_\nu(G_i))$ of an IFG is given by the euations:

$$LE(A_\mu(G_i)) = \sum_{i=1}^{n} \left| \lambda_i - \frac{2\sum_{1\le i\le j\le n} \mu(v_i, v_j)}{n} \right|,$$

$$LE(A_\nu(G_i)) = \sum_{i=1}^{n} \left| \theta_i - \frac{2\sum_{1\le i\le j\le n} \nu(v_i, v_j)}{n} \right|.$$

Definition 7. [Correlation coefficient of IFGs] The Intuitionistic energies of two Intuitionistic Fuzzy Graphs G_1 and G_2 are described as

$$E_{IFG}(G_1) = \sum_{i=1}^{n} \left[\mu_{G_1}^2(x_i) + \nu_{G_1}^2(x_i)\right] = \sum_{j=1}^{n} \lambda_j^2(G_1)$$

and

$$E_{IFG}(G_2) = \sum_{i=1}^{n} \left[\mu_{G_2}^2(x_i) + \nu_{G_2}^2(x_i)\right] = \sum_{j=1}^{n} \lambda_j^2(G_2).$$

The covariance of the IFGs G_1 and G_2 is defined as

$$C_{IFG}(G_1, G_2) = \sum_{i=1}^{n} \left[\mu_{G_1}(x_i)\mu_{G_2}(x_i) + \nu_{G_1}(x_i)\nu_{G_2}(x_i)\right].$$

Therefore, the correlation coefficient measure of IFGs G_1 and G_2 are given by the equation

$$K_{IFG}(G_1,G_2) = \frac{C_{IFG}(G_1,G_2)}{\sqrt{E_{IFG}(G_1)E_{IFG}(G_2)}} = \frac{\sum_{i=1}^{n}\left[\mu_{G_1}(x_i)\mu_{G_2}(x_i)+\nu_{G_1}(x_i)\nu_{G_2}(x_i)\right]}{\sqrt{\sum_{i=1}^{n}\left[\mu_{G_1}^2(x_i)+\nu_{G_1}^2(x_i)\right]}\sqrt{\sum_{i=1}^{n}\left[\mu_{G_2}^2(x_i)+\nu_{G_2}^2(x_i)\right]}}$$

Alternately, Xu et al., developed an alternate version of the CC of IFGs C and D, so the same form can be converted on IFGs G_1 and G_2 as follows.

$$K_{IFG}(G_1,G_2) = \frac{\sum_{i=1}^{n}\left[\mu_{G_1}(x_i)\mu_{G_2}(x_i)+\nu_{G_1}(x_i)\nu_{G_2}(x_i)\right]}{Max\left\{\left[\sum_{i=1}^{n}\left[\mu_{G_1}^2(x_i)+\nu_{G_1}(x_i)\right]\right]^{\frac{1}{2}},\left[\sum_{i=1}^{n}\left[\mu_{G_2}^2(x_i)+\nu_{G_2}^2(x_i)\right]\right]^{\frac{1}{2}}\right\}}$$

or

$$K_{IFG}(G_1,G_2) = \frac{\sum_{i=1}^{n}\left[\mu_{G_1}(x_i)\mu_{G_2}(x_i)+\nu_{G_1}(x_i)\nu_{G_2}(x_i)+\pi_{G_1}(x_i)\pi_{G_2}(x_i)\right]}{Max\left\{\left[\sum_{i=1}^{n}\left[u_{G_1}^2(x_i)+\nu_{G_1}^2(x_i)+\pi_{G_1}^2(x_i)\right]\right]^{\frac{1}{2}},\left[\sum_{i=1}^{n}\left[\mu_{G_2}^2(x_i)+\nu_{G_2}^2(x_i)+\pi_{G_2}^2(x_i)\right]\right]^{\frac{1}{2}}\right\}}$$

or

$$K_{IFG}(G_1,G_2) = \frac{\sum_{i=1}^{n}\left[\mu_{G_1}(x_i)\mu_{G_1}(x_i)+\nu_{G_1}(x_i)\nu_{G_2}(x_i)+\pi_{G_1}(x_i)\pi_{G_2}(x_i)\right]}{\left\{\sqrt{\sum_{i=1}^{n}\left[\mu_{G_1}^2(x_i)+\nu_{G_1}^2(x_i)+\pi_{G_1}^2(x_i)\right]}\sqrt{\sum_{i=1}^{n}\left[\mu_{G_2}^2(x_i)+\nu_{G_2}^2(x_i)+\pi_{G_2}^2(x_i)\right]}\right\}}.$$

The function K_{IFG} satisfies the following conditions

- (P_1): $0 \le K_{IFG}(G_1,G_2) \le 1$,
- (P_2): $K_{IFG}(G_1,G_2) = K_{IFG}(G_1,G_2)$,
- (P_3): $K_{IFG}(G_1,G_2) = 1$, if $G_1 = G_2$.

3 GROUP DECISION-MAKING BASED ON INTUITIONISTIC FUZZY GRAPHS LAPLACIAN ENERGY AND CORRELATION COEFFICIENT

3.1 Algorithm

For the purpose of finding GDMP based on IFPR, let $\omega = (\omega_1, \omega_2, \ldots, \omega_m)$ be a subjective loading vector of authorities, where $\omega_k > 0,\ k = 1, 2, \ldots, m\ with\ \sum_{i=1}^{m} \omega_i = 1$.

Step (i). Calculate the $LE(G_i)$ using the following equations.

$$LE(G_i) = \sum_{i=1}^{n} \left| \lambda_i - \frac{2\sum_{1\le i\le j\le n} \mu(v_i, v_j)}{n} \right|,$$
$$LE(G_i) = \sum_{i=1}^{n} \left| \theta_i - \frac{2\sum_{1\le i\le j\le n} \nu(v_i, v_j)}{n} \right|. \tag{1}$$

Step (ii). Calculate the weight ω_k^a by using Laplacian energy of the authorities e_k using the equation

$$\omega_k^a = ((\omega_\mu)_k, (\omega_\nu)_k) = \left[\frac{LE((G_\mu)_k)}{\sum_{i=1}^{m} LE((G_\mu)_i)}, \frac{LE((G_\nu)_k)}{\sum_{i=1}^{m} LE((G_\nu)_i)} \right]. \tag{2}$$

Step (iii). Calculate the Karl Pearson's correlation coefficient $K(G_s, G_l)$ between G_s and G_l for $s \neq l$, using the equation

$$K_{IFG}(G_s, G_l) = \frac{\sum_{i=1}^{n} \left[\mu_{G_s}(x_i)\mu_{G_l}(x_i) + \nu_{G_s}(x_i)\nu_{G_l}(x_i)\right]}{\sqrt{\sum_{i=1}^{n} \left[\mu_{G_s}^2(x_i) + \nu_{G_s}^2(x_i)\right]}\sqrt{\sum_{i=1}^{n} \left[\mu_{G_l}^2(x_i) + \nu_{G_l}^2(x_i)\right]}}. \tag{3}$$

Compute the average correlation coefficient degree $K(G_s)$ to the others by using the equation

$$K(G_s) = \frac{1}{m-1} \sum_{l=1, s\neq l}^{m} K(G_s, G_l), \quad s = 1, 2, \ldots, m. \tag{4}$$

Step (iv). Compute the weight ω_s^b determined by $K(G_s)$ of the authority e_k, using the equation

$$\omega_s^b = \frac{K(G_s)}{\sum_{i=1}^{m} K(G_i)}, \quad s = 1, 2, \ldots, m. \tag{5}$$

Step (v). Calculate the authority $e_k's$ objective weight ω_s^2 using the following equation

$$\omega_s^2 = \eta\, \omega_s^a + (1-\eta)\, \omega_s^b, \quad \eta \in [0, 1], \quad s = 1, 2, \ldots, m. \tag{6}$$

Step (vi). Incorporate the weight ω_s with authority e_k subjective weight ω_s^a and objective weight ω_s^2 using the equation

$$\omega_s = \gamma\omega_s^1 + (1-\gamma)\omega_s^2, \quad \gamma \in [0,1], \quad s = 1,2,\ldots,m. \tag{7}$$

3.2 Procedure – I

Step (vii). Use the equation

$$\tau_i^{(s)} = \frac{1}{n}\sum_{j=1}^{n} \tau_{ij}^{(s)}, \tag{8}$$

where $i = 1,2,\ldots,m$, to obtain the aggregate intuitionistic ambiguity value of the option $\tau_i^{(s)}$ across all alternatives.

Step (viii). Use the equation

$$\tau_i = \sum_{i=1}^{m} \omega_s \tau_i^{(s)}, \quad \forall i = 1,2,\ldots,m \tag{9}$$

to make a total intuitionistic ambiguity value of the alternative τ_i over other choices by summing all $\tau_i^{(s)}$ $(s = 1,2,\ldots,n)$, corresponding to n-authorities.

Step (ix). Calculate the rank function from the equation

$$K(\tau_i) = \mu_i - \nu_i \tag{10}$$

of τ_i if the better value of $K(\tau_i)$ is the finer alternate τ_i, then the alternates must be ranked in groups.

3.3 Procedure – II

Step (i). Determine the supportive IFPR as $M = (\tau_{ij})_{n\times n}$ by the equation

$$\tau_{ij} = (\mu_{ij}, \nu_{ij}) = \left(\sum_{l=1}^{m} \omega_l \mu_{ij}^{(l)}, \sum_{l=1}^{m} \omega_l \nu_{ij}^{(l)}\right), \quad i,j = 1,2,\ldots,n. \tag{11}$$

Step (ii). For every choice x_i, decide the correlation coefficient value $K(M^i, M^+)$ between M^i and M^+ and the correlation coefficient value $K(M^i, M^-)$ between M^i and M^- using the equations

$$K(M^i, M^+) = \frac{1}{n}\sum_{j=1}^{n} \frac{\mu_{ij}(1) + \nu_{ij}(0)}{\sqrt{\mu_{ij}^2 + \nu_{ij}^2}\sqrt{1^2 + 0^2}} = \frac{1}{n}\sum_{j=1}^{n} \frac{\mu_{ij}}{\sqrt{\mu_{ij}^2 + \nu_{ij}^2}} \tag{12}$$

and

$$K(M^i, M^-) = \frac{1}{n}\sum_{j=1}^{n} \frac{\mu_{ij}(0) + \nu_{ij}(1)}{\sqrt{\mu_{ij}^2 + \nu_{ij}^2}\sqrt{0^2 + 1^2}} = \frac{1}{n}\sum_{j=1}^{n} \frac{\nu_{ij}}{\sqrt{\mu_{ij}^2 + \nu_{ij}^2}}. \tag{13}$$

Step (iii). For each choice x_i, ascertain its estimate value by the equation

$$h(x_i) = \frac{K(M^i, M^+)}{K(M^i, M^+) + K(M^i, M^-)}. \tag{14}$$

The two procedures (I and II) listed above are intended for acquiring the included loads and ranking the substitutes. When the value of $h(x_i)$ is greater, the alternative x_i is preferred. The finest ranking of the substitutes is then available for decision-makers.

4 FLOW CHART

The flowchart below illustrates how the suggested technique would work to get the alternate rankings.

5 APPLICATION: FINEST SELECTION OF MONEY-INVESTING SCHEMES

Suppose a man who wants to invest his money in any of the four categories such as Fixed deposit (F_D, x_1), Govt bonds (G_B, x_2), Postal savings (P_S, x_3), and Shares (S_H, x_4) (Wang et al. 2005) [32]. He can only pick one based on three criteria such as Tax benefits (e_1), Risk coverage (e_2) and Rate of interest (e_3). Due to his inadequate expertise, he wanted to seek advice from experts who could offer the finest investment strategy. As a result, the experts will apply IFGs to express their preference ratings in order to find the original ranking information, which is provided in the intuitionistic fuzzy decision matrices. It should be noted that the criteria are classified into two types:

1. Benefit type and
2. Price type.

This should be considered by the experts and client when selecting preference values.

To determine one of the most desired categories, the recommended experts use the appropriate aggregate decision information. In order to choose the best category, they use the correlation coefficient and LE of IGFs based on GDMP as follows.

From Figure 2, the IFAM is defined as

$$A(G_1) = \begin{bmatrix} (0,0) & (0.2,0.4) & (0.5,0.4) & (0.7,0.1) \\ (0.4,0.2) & (0,0) & (0.3,0.5) & (0.4,0.5) \\ (0.4,0.5) & (0.5,0.3) & (0,0) & (0.8,0.2) \\ (0.1,0.7) & (0.5,0.4) & (0.2,0.8) & (0,0) \end{bmatrix}.$$

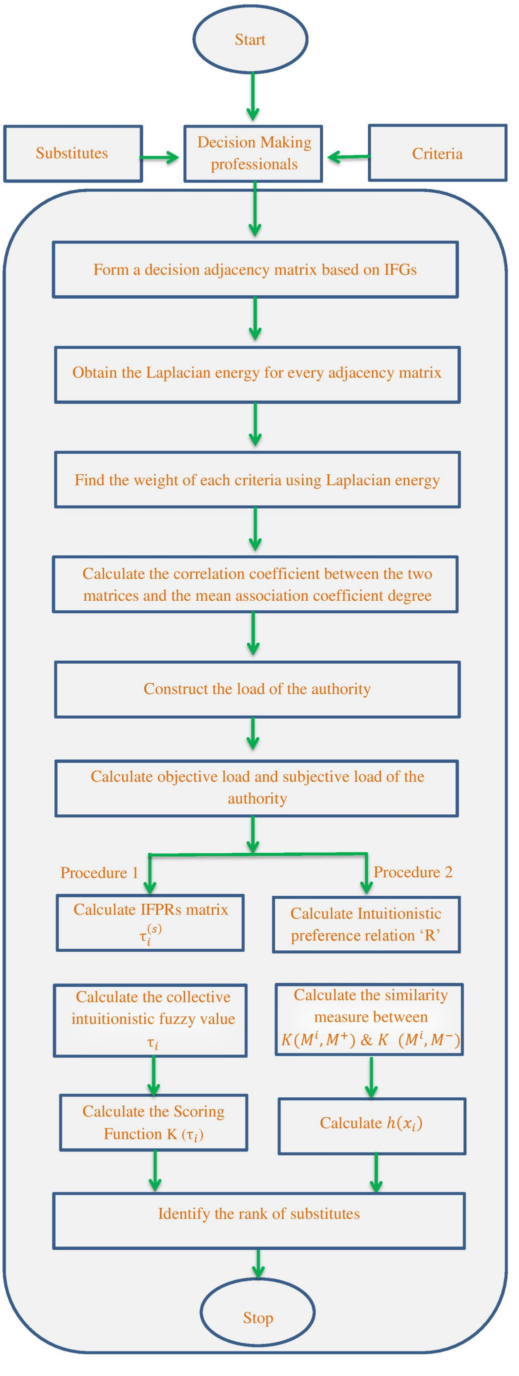

Figure 1. The procedure of ranking the alternatives (substitutes) for GDM assessment

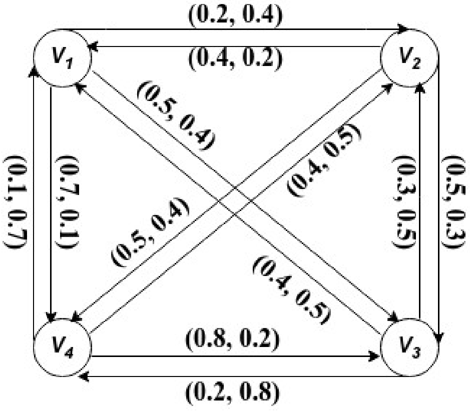

Figure 2. IFG (G_1) related to tax benefits

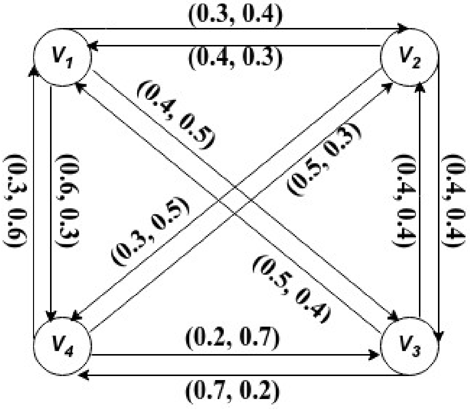

Figure 3. IFG (G_2) related to risk coverage

From Figure 3, the IFAM is defined as

$$A(G_2) = \begin{bmatrix} (0,0) & (0.3,0.4) & (0.4,0.5) & (0.6,0.3) \\ (0.4,0.3) & (0,0) & (0.4,0.4) & (0.5,0.3) \\ (0.5,0.4) & (0.4,0.4) & (0,0) & (0.7,0.2) \\ (0.3,0.6) & (0.3,0.5) & (0.2,0.7) & (0,0) \end{bmatrix}.$$

From Figure 4, the IFAM is defined as

$$A(G_3) = \begin{bmatrix} (0,0) & (0.8,0.1) & (0.3,0.4) & (0.6,0.4) \\ (0.1,0.8) & (0,0) & (0.5,0.3) & (0.4,0.5) \\ (0.4,0.3) & (0.3,0.5) & (0,0) & (0.3,0.7) \\ (0.4,0.6) & (0.5,0.4) & (0.7,0.3) & (0,0) \end{bmatrix}.$$

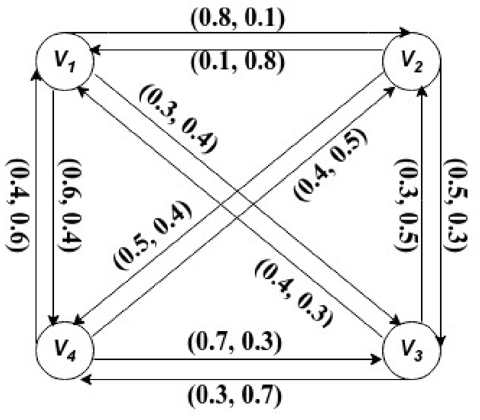

Figure 4. IFG (G_3) related to rate of interest

The Laplacian IFAM $A(G_1)$ of G_1 is given by

$$L(A(G_1)) = D(G_1) - A(G_1),$$

$$L(A(G_1)) = \begin{bmatrix} (1.4, 0.9) & (0,0) & (0,0) & (0,0) \\ (0,0) & (1.1, 1.2) & (0,0) & (0,0) \\ (0,0) & (0,0) & (1.7, 1.0) & (0,0) \\ (0,0) & (0,0) & (0,0) & (0.8, 1.9) \end{bmatrix}$$

$$- \begin{bmatrix} (0,0) & (0.2, 0.4) & (0.5, 0.4) & (0.7, 0.1) \\ (0.4, 0.2) & (0,0) & (0.3, 0.5) & (0.4, 0.5) \\ (0.4, 0.5) & (0.5, 0.3) & (0,0) & (0.8, 0.2) \\ (0.1, 0.7) & (0.5, 0.4) & (0.2, 0.8) & (0,0) \end{bmatrix}.$$

The Laplacian IFAM $A(G_2)$ of G_2 is

$$L(A(G_2)) = D(G_2) - A(G_2),$$

$$L(A(G_2)) = \begin{bmatrix} (1.3, 1.2) & (0,0) & (0,0) & (0,0) \\ (0,0) & (1.3, 1.0) & (0,0) & (0,0) \\ (0,0) & (0,0) & (1.6, 1.0) & (0,0) \\ (0,0) & (0,0) & (0,0) & (0.8, 1.8) \end{bmatrix}$$

$$- \begin{bmatrix} (0,0) & (0.3, 0.4) & (0.4, 0.5) & (0.6, 0.3) \\ (0.4, 0.3) & (0,0) & (0.4, 0.4) & (0.5, 0.3) \\ (0.5, 0.4) & (0.4, 0.4) & (0,0) & (0.7, 0.2) \\ (0.3, 0.6) & (0.3, 0.5) & (0.2, 0.7) & (0,0) \end{bmatrix}.$$

The Laplacian IFAM $A(G_3)$ of G_3 is

$$L(A(G_3)) = D(G_3) - A(G_3),$$

$$L(A(G_3)) = \begin{bmatrix} (1.7,0.9) & (0,0) & (0,0) & (0,0) \\ (0,0) & (1.0,1.6) & (0,0) & (0,0) \\ (0,0) & (0,0) & (1.0,1.5) & (0,0) \\ (0,0) & (0,0) & (0,0) & (1.6,1.3) \end{bmatrix}$$

$$- \begin{bmatrix} (0,0) & (0.8,0.1) & (0.3,0.4) & (0.6,0.4) \\ (0.1,0.8) & (0,0) & (0.5,0.3) & (0.4,0.5) \\ (0.4,0.3) & (0.3,0.5) & (0,0) & (0.3,0.7) \\ (0.4,0.6) & (0.5,0.4) & (0.7,0.3) & (0,0) \end{bmatrix}.$$

5.1 Algorithm

Step (i). By formula 1, we calculate the LEs of G_i, $i = 1, 2, 3$.

From Figure 2 and $A(G_1)$ we get

$$LE(G_1) = (2.5796, 2.7298).$$

From Figure 3 and $A(G_2)$ we get

$$LE(G_2) = (2.5000, 2.5000).$$

From Figure 4 and $A(G_3)$ we get

$$LE(G_3) = (2.7425, 2.7047).$$

Step (ii). Using formula 2, we get the weights of G_i determined with LEs as follows:

$$\omega_1^a = (0.3298, 0.3440),$$
$$\omega_2^a = (0.3196, 0.3151)$$

and

$$\omega_3^a = (0.3506, 0.3409).$$

Step (iii). Using 3 formula, we have

$$K(G_1, G_2) = 0.9681,$$
$$K(G_1, G_3) = 0.7794$$

and

$$K(G_2, G_3) = 0.8350.$$

By Equation (4), we get

$$K(G_1) = 0.8738,$$
$$K(G_2) = 0.9016$$

and

$$K(G_3) = 0.8072.$$

Step (iv). By Equation (5), we have $\omega_s^b = \frac{K(G_s)}{\sum_{i=1}^{m} K(G_i)}$, $s = 1, 2, \ldots, m$. then we get

$$\omega_1^b = 0.3383,$$
$$\omega_2^b = 0.3491$$

and

$$\omega_3^b = 0.3126.$$

Step (v). By Equation (6), we have $\omega_s^2 = \eta\omega_s^a + (1-\eta)\omega_s^b$, and taking $\eta = 0.5$ we get

$$\omega_{1,\mu}^2 = 0.3341,$$
$$\omega_{2,\mu}^2 = 0.3344,$$
$$\omega_{3,\mu}^2 = 0.3316$$

and

$$\omega_{1,\nu}^2 = 0.3412,$$
$$\omega_{2,\nu}^2 = 0.3321,$$
$$\omega_{3,\nu}^2 = 0.3268.$$

So, weights of authorities are

$$\omega_1^2 = (0.3341, 0.3412),$$
$$\omega_2^2 = (0.3344, 0.3321)$$

and

$$\omega_3^2 = (0.3316, 0.3268).$$

Step (vi). By Equation (7), we have $\omega_s = \gamma\omega_s^a + (1-\gamma)\omega_s^2$ and taking $\gamma = 0.5$ we get

$$\omega_{1,\mu} = 0.3320,$$
$$\omega_{2,\mu} = 0.3270,$$
$$\omega_{3,\mu} = 0.3411$$

and

$$\omega_{1,\nu} = 0.3426,$$
$$\omega_{2,\nu} = 0.3236,$$
$$\omega_{3,\nu} = 0.3339.$$

So, the impartial weights are

$$\omega_1 = (0.3320, 0.3426),$$
$$\omega_2 = (0.3270, 0.3236)$$

and

$$\omega_3 = (0.3411, 0.3339).$$

5.2 Procedure I

Step (vii). By Equation (8), we have $\tau_i^{(s)} = \frac{1}{n}\sum_{j=1}^{n} \tau_{ij}^{(s)}$, $i = 1, 2, \ldots, m$.

Then from Figure 2 and $A(G_1)$ we get

$$\tau_1^{(1)} = (0.4667, 0.3000),$$
$$\tau_2^{(1)} = (0.3667, 0.4000),$$
$$\tau_3^{(1)} = (0.5667, 0.3334),$$
$$\tau_4^{(1)} = (0.2667, 0.6334).$$

From Figure 3 and $A(G_2)$ we get

$$\tau_1^{(2)} = (0.4334, 0.4000),$$
$$\tau_2^{(2)} = (0.4334, 0.3334),$$
$$\tau_3^{(2)} = (0.5334, 0.3334),$$
$$\tau_4^{(2)} = (0.2667, 0.6000).$$

From Figure 4 and $A(G_3)$ we get

$$\tau_1^{(3)} = (0.5667, 0.3000),$$
$$\tau_2^{(3)} = (0.3334, 0.5334),$$
$$\tau_3^{(3)} = (0.3334, 0.5000),$$
$$\tau_4^{(3)} = (0.5334, 0.4334).$$

Step (viii). By Equation (9), we have $\tau_i = \sum_{s=1}^{m} \omega_s \tau_i^{(s)}$, $i = 1, 2, \ldots, n.$, we get

$$\tau_{1,\mu} = 0.4900, \qquad \tau_{1,\nu} = 0.3324,$$
$$\tau_{2,\mu} = 0.3772, \qquad \tau_{2,\nu} = 0.4230,$$
$$\tau_{3,\mu} = 0.4763, \qquad \tau_{3,\nu} = 0.3891$$

and

$$\tau_{4,\mu} = 0.3577, \qquad \tau_{4,\nu} = 0.5559.$$

Therefore

$$\tau_1 = (0.4900, 0.3324),$$
$$\tau_2 = (0.3772, 0.4230),$$
$$\tau_3 = (0.4763, 0.3891)$$

and

$$\tau_4 = (0.3577, 0.5559).$$

Step (ix). By Equation (10), we have $K(\tau_i) = \mu_i - \nu_i$, we get

$$K(\tau_1) = 0.1576,$$
$$K(\tau_2) = -0.0450,$$
$$K(\tau_3) = 0.0872,$$
$$K(\tau_4) = -0.1982.$$

Therefore $K(\tau_1) > K(\tau_3) > K(\tau_2) > K(\tau_4)$, as a result $\tau_1 > \tau_3 > \tau_2 > \tau_4$.

The resulting ranking order is the same for all the values of γ ($\gamma \in [0, 1]$), not only the one ($\gamma = 0.5$) used in Equation (7).

5.3 Procedure II

Step (i). In this part, we present the position outcome potential using our comparable correlation coefficient approach. By Equation (11) in method II, we form the group IFPR as follows.

From the matrices $A(G_1)$, $A(G_2)$ and $A(G_3)$ we get

$$M = \begin{bmatrix} (0,0) & (0.4376, 0.2999) & (0.3994, 0.4324) & (0.6333, 0.2649) \\ (0.2977, 0.4327) & (0,0) & (0.4010, 0.4009) & (0.4327, 0.4353) \\ (0.4327, 0.4009) & (0.3991, 0.3992) & (0,0) & (0.6309, 0.3670) \\ (0.2677, 0.6343) & (0.4347, 0.4324) & (0.3706, 0.6008) & (0,0) \end{bmatrix}.$$

Step (ii). By using the Equations (12) and (13), we achieve

$$K(M^1, M^+) = 0.6065,$$
$$K(M^2, M^+) = 0.4947,$$
$$K(M^3, M^+) = 0.5762,$$
$$K(M^4, M^+) = 0.4057$$

and

$$K(M^1, M^-) = 0.4215,$$
$$K(M^2, M^-) = 0.5601,$$
$$K(M^3, M^-) = 0.4724,$$
$$K(M^4, M^-) = 0.6194.$$

Step (iii). Next, for each choice x_i, $(i = 1, 2, 3, 4)$, Equation (14) provides the computation standards as

$$h(x_1) = 0.5900,$$
$$h(x_2) = 0.4690,$$
$$h(x_3) = 0.5494,$$
$$h(x_4) = 0.3958.$$

Since $h(x_1) > h(x_3) > h(x_2) > h(x_4)$, as a result $x_1 > x_3 > x_2 > x_4$.

The resulting ranking order is the same for all the values γ, where $\gamma \in [0, 1]$.

According to $Xu's$ algorithm [33] with Procedures I and II, rank wise Fixed deposit (x_1) is at the top position, Shares (x_4) are at the last, and Govt bonds (x_2) and Postal savingas (x_3) are in the middle position. Also,the position ordering of alternatives is the same for both procedures and are shown in the following tables.

After the assessment, the decision-maker concludes that a fixed deposit is the best option for a person looking to invest money among the four categories mentioned. The overall analysis revealed that the two working methods produced the same ranking order. Furthermore, when compared to the method (see [22]), this approach yields slightly faster results.

γ	$\boldsymbol{\omega}$	$\boldsymbol{\tau}$
0.3	$\omega_1 = (0.3328, 0.3420)$	$\tau_1 = (0.4894, 0.3327)$
	$\omega_2 = (0.3298, 0.3270)$	$\tau_2 = (0.3774, 0.4662)$
	$\omega_3 = (0.3373, 0.3310)$	$\tau_3 = (0.4770, 0.3885)$
		$\tau_4 = (0.3566, 0.5568)$
0.5	$\omega_1 = (0.3320, 0.3426)$	$\tau_1 = (0.4900, 0.3324)$
	$\omega_2 = (0.3270, 0.3236)$	$\tau_2 = (0.3772, 0.4230)$
	$\omega_3 = (0.3411, 0.3339)$	$\tau_3 = (0.4763, 0.3891)$
		$\tau_4 = (0.3577, 0.5559)$
0.7	$\omega_1 = (0.3311, 0.3432)$	$\tau_1 = (0.4904, 0.3321)$
	$\omega_2 = (0.3240, 0.3202)$	$\tau_2 = (0.3768, 0.4450)$
	$\omega_3 = (0.3449, 0.3367)$	$\tau_3 = (0.4754, 0.3895)$
		$\tau_4 = (0.3587, 0.5554)$

Table 2. The table values of the alternatives for distinct values of γ using Xu's technique and working procedure I

γ	$\mathbf{K}(\tau_1)$	$\mathbf{K}(\tau_2)$	$\mathbf{K}(\tau_3)$	$\mathbf{K}(\tau_4)$	**Ranking**
0.3	0.1567	-0.0888	0.0885	-0.2002	$\tau_1 > \tau_3 > \tau_2 > \tau_4$
0.5	0.1576	-0.0450	0.0872	-0.1982	$\tau_1 > \tau_3 > \tau_2 > \tau_4$
0.7	0.1583	-0.0682	0.0859	-0.1967	$\tau_1 > \tau_3 > \tau_2 > \tau_4$

Table 3. The ranking order of the alternatices by using Xu's technique and working procedure I

γ	$\boldsymbol{\omega}$	$\mathbf{K}(\mathbf{M^i}, \mathbf{M^+})$	$\mathbf{K}(\mathbf{M^i}, \mathbf{M^-})$
0.3	$(0.3328, 0.3420)$	$K(M^1, M^+) = 0.6060$	$K(M^1, M^-) = 0.4222$
	$(0.3298, 0.3270)$	$K(M^2, M^+) = 0.4954$	$K(M^2, M^-) = 0.5596$
	$(0.3373, 0.3310)$	$K(M^3, M^+) = 0.5736$	$K(M^3, M^-) = 0.4769$
		$K(M^4, M^+) = 0.4047$	$K(M^4, M^-) = 0.6201$
0.5	$(0.3320, 0.3426)$	$K(M^1, M^+) = 0.6065$	$K(M^1, M^-) = 0.4215$
	$(0.3270, 0.3236)$	$K(M^2, M^+) = 0.4947$	$K(M^2, M^-) = 0.5601$
	$(0.3411, 0.3339)$	$K(M^3, M^+) = 0.5762$	$K(M^3, M^-) = 0.4724$
		$K(M^4, M^+) = 0.4057$	$K(M^4, M^-) = 0.6194$
0.7	$(0.3311, 0.3432)$	$K(M^1, M^+) = 0.6068$	$K(M^1, M^-) = 0.4210$
	$(0.3240, 0.3202)$	$K(M^2, M^+) = 0.4940$	$K(M^2, M^-) = 0.5606$
	$(0.3449, 0.3367)$	$K(M^3, M^+) = 0.5593$	$K(M^3, M^-) = 0.4785$
		$K(M^4, M^+) = 0.4067$	$K(M^4, M^-) = 0.6188$

Table 4. The table values of the replacements for distinct values of γ using Xu's technique and working procedure II

γ	$\mathbf{K}(\tau_1)$	$\mathbf{K}(\tau_2)$	$\mathbf{K}(\tau_3)$	$\mathbf{K}(\tau_4)$	**Ranking Order**
0.3	0.5894	0.4696	0.5460	0.3949	$x_1 > x_3 > x_2 > x_4$
0.5	0.5900	0.4690	0.5494	0.3958	$x_1 > x_3 > x_2 > x_4$
0.7	0.5904	0.4684	0.5389	0.3966	$x_1 > x_3 > x_2 > x_4$

Table 5. The ranking order of the replacements for distinct values of γ using Xu's technique and working procedure II

6 CONCLUSION

In general, the opinions of the authorities on alternatives might be unclear and divergent when there is a lack of information or expertise concerning an ambiguous situation. The ideal solution to this issue is the intuitionistic fuzzy concept. In this paper, we illustrated how correlation coefficient measures and Laplacian energy can be used to solve GDM problems when the weight of the criterion is completely unknown and the IFG is the main factor that affects the alternatives. The proposed statistical measure has been successfully implemented for money-investing schemes, and its use will aid in ranking the substitutes. This analogous approach can be used to investigate other aspects of various fuzzy graphs and is also applicable to many IFG types, including Hesitancy fuzzy graphs, Complex fuzzy graphs, etc.

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