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MULTI-OBJECTIVE OPTIMIZATION FOR MULTI-MODAL ROUTE PLANNING INTEGRATING SHARED TAXI AND BUS

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Abstract. Multi-modal transportation, emerging as a sustainable travel option, has shown immense promise in reducing passengers' travel expenses and vehicles' energy consumption. To further promote green travel, this work studies a multi-modal route planning problem, focusing on the integration of shared taxis and buses. The objective is to devise an innovative route planning approach for shared taxis, enabling passengers a seamless transition between the two modes and arrive at their destinations within designated timeframes. It designs a new pricing rule and establishes a multi-objective optimization that takes into account both the interests of passengers and shared taxi operators. The objectives are to minimize the aggregate cost incurred by all passengers and the overall travel distance traversed by shared taxis, and maximizes the revenue earned per kilometer by shared taxi operators. A novel nondominated linear sorting genetic algorithm (NLSGA) is introduced to tackle the problem. This algorithm incorporates innovative evolution and selection strategies to preserve solution diversity and enhance convergence speed. NLSGA demonstrates superior performance compared to several widely used multi-objective optimization algorithms, including NSGA-II, MOPSO, and MOGWO. Experimental results reveal that the proposed algorithm effectively reduces passengers' cost and shared taxis' travel distance while simultaneously maximizing revenue per kilometer for shared taxi operators.

Keywords: Multi-modal transportation, multi-modal route planning problem, multi-objective optimization, nondominated linear sorting genetic algorithm

Mathematics Subject Classification 2010: 90C27, 90C29

1 INTRODUCTION

In recent years, the rapid increase in private vehicles has resulted in urban congestion and deteriorating air quality. Urgent measures are required to reduce private vehicles usage or explore alternative transportation modes [1]. Public transport, promoted by the government, is a sustainable travel option that can help address some of the issues caused by private vehicles [2]. However, improvements are needed in the flexibility, comfort, and availability of public transport, as it is limited by fixed routes [3]. Hence, it is worthwhile to explore multi-modal transportation, which integrates different modes of transport (such as walking, car travel, and public transport), offering an environmentally friendly and efficient travel solution [4, 5].

Numerous studies explore the integration of (non-ridesharing) vehicles with public transport [6, 7, 8]. Some propose coordinating ride-hailing (or ride-sourcing) services, which leverage online platforms to provide users with convenient and personalized travel options [9, 10, 11]. For example, Feng et al. design an efficient real-time order dispatching method to coordinate ride-sourcing (ride-hailing) and public transport service with reinforcement learning [9]. Others focus on integrating (non-ridesharing) vehicles with public transport based on specific design objectives, such as minimizing the travel time of passengers [12] and minimizing the total travel cost and waiting time experienced by passengers [13]. Nevertheless, researchers are concerned that the introduction of vehicles without ridesharing may exacerbate traffic congestion, as it could attract demand from other modes of transportation, while the need for vehicle relocations would also result in additional mileage [12].

Many scholars propose that ridesharing has the potential to enhance seat utilization, alleviate traffic congestion, and reduce cost and energy consumption [14].

Ridesharing refers to a transportation mode where individual travelers share a vehicle for a trip, splitting travel costs, including gas, toll, and parking fees with others that have similar itineraries and time schedules [15]. Therefore, some studies consider ridesharing as a feeder service, which is a solution to the first/last mile problem in accessing public transport. For instance, Yap et al. conduct a stated preference experiment using a discrete choice model to explore the feasibility of ridesharing for last-mile trips between train stations and travelers' final destinations [16]. Zubin et al. employ a stakeholder survey to establish a range of scenarios for introducing driverless shuttles as a first/last-mile option in multi-modal journeys [17]. In addition, different objectives are considered when determining ridesharing strategies, and intelligent optimization algorithms. Typical objectives include maximizing the number of matched riders [18, 19], maximizing the total served passengers [20], and minimizing the total increase in driving distance for all drivers [19]. However, these passengers are generally directed to the fixed or nearest public transport transfer stations [20, 21, 22].

Furthermore, some studies propose the integration of ridesharing with public transport, where each passenger's travel mode is not fixed, and the public transport transfer stations are uncertain, posing additional complexities to the ridesharing matching problem. Various studies have been conducted on it. For example, Huang et al. create a multi-modal network by merging the public transport time-extended model and the carpooling offers (typically consists of origins, destinations, and a set of stopover points along the way) time-extended model [23]. Lau and Susilawati develop a multi-modal transportation system by connecting public transport with pre-assigned vehicle routes between stations, while considering the passengers' preferences for travel modes [24]. However, they do not take into account the factor of passengers' walking. Additionally, these studies generally have the following limitations. Firstly, they primarily focus on the interests of passengers, such as minimizing the total detour time of the passengers [25], and minimizing the access time of passengers to idle vehicles [26], while ignoring the total benefits of vehicles. Therefore, we aim to maximize the interests of both passengers and vehicles simultaneously. Secondly, the consideration of the pricing rule is incomprehensive. The discounts for passenger walking and additional seat charges for multiple passengers within the same request are not taken into consideration in their research.

To overcome these shortcomings, this paper develops a multi-modal transportation system that integrates shared taxis and buses, and presents a multi-modal route planning problem that encompasses route planning for shared taxis and trip planning for passengers. Our approach focuses on optimizing trip planning for each passenger, which reduces the number of transfers and ensures passengers reach their destinations within designated timeframes. Additionally, we develop the optimal route planning for shared taxis, aiming to reduce the travel distance of shared taxis and enhance the revenue per kilometer.

In parallel, intelligent optimization algorithms, such as ant colony optimization algorithm [27] and non-dominated sorting genetic algorithm II (NSGA-II) [28], are widely applied to combinatorial optimization problems. For example, Huang et al. propose an ant path-oriented carpooling allocation approach to solve the carpool service problem with time windows [29]. Duan et al. develop a robust multi-objective particle swarm optimizer to solve the vehicle routing problem with time windows [30]. Seo and Asakura tackle a multi-objective linear optimization problem that jointly optimizes aggregated variables on shared autonomous vehicle's routing and passenger pickup/delivery [31]. He et al. develop a tailored adaptive large neighborhood search algorithm with an accelerated strategy for obtaining robust near-optimal solutions within a reasonable time [32]. However, these optimization algorithms primarily focus on solving the ridesharing service problem and do not address the multi-modal route planning problem for shared taxis and buses. To fill this gap, this paper introduces a novel non-dominated linear sorting genetic algorithm (NLSGA), enabling comprehensive scheduling for shared taxis and trip planning for passengers.

The main contributions of this paper can be summarized as follows:

1. It designs a combined travel mode which is a multi-modal transportation that combines shared taxis and buses, and it establishes a multi-objective optimization model aiming at reducing cost for passengers while ensuring their timely arrival at destinations and increasing the revenue per kilometer of shared taxis. At the same time, a new pricing rule is designed to support this combined travel mode.
2. It proposes the NLSGA to address the multi-modal route planning problem. A new evolution strategy is designed for the selection of pick-up and drop-off stations, and three selection strategies are designed to speed up the search for solutions. The results illustrate the NLSGA outperforms several existing optimization algorithms in solving this problem.

The remainder of this paper is organized as follows. Section 2 formally defines the problem and constructs a multi-objective optimization model. Section 3 introduces a solution approach. Section 4 performs numerical experiments and discusses the relevant results. Finally, in Section 5, we conclude this paper and discuss the future work.

2 PROBLEM DESCRIPTION AND FORMULATION

This section introduces the multi-modal route planning problem and formulates its mathematical model.

2.1 Preliminaries

A set of all requests is denoted by I, a set of all shared taxis is denoted by J, and a set of all bus lines is denoted by L. A request i is defined as $r_i = (o_i, d_i, t_i, a_i, c_i)$, where o_i is its origin, d_i is its destination, t_i is its dispatch time, a_i is its latest arrival time, and c_i is the number of passengers of r_i. Let s_j be shared taxi j, b_l be bus line l. All shared taxis are parked in different parking lots. Each request is served by at most one bus and has a pair of pick-up and drop-off stations at most. Let $\check{k}_i$ and $\hat{k}_i$ be the pick-up and drop-off stations of r_i, respectively. The notations used in the paper are presented in Table 1.

I	A set of all requests.
J	A set of shared taxis.
L	A set of bus lines.
r_i	Request i, $r_i = (o_i, d_i, t_i, a_i, c_i)$, $i \in I$.
o_i	Origin of r_i.
d_i	Destination of r_i.
t_i	Dispatch time of r_i.
a_i	Latest arrival time of r_i.
c_i	Number of passengers of r_i.
s_j	Shared taxi j, $j \in J$.
b_l	Bus l, $l \in L$.
$\check{k}_i$	Pick-up station of r_i.
$\hat{k}_i$	Drop-off station of r_i.
Ω	A set of all points.
u, v	Two points in Ω.
$D(u, v)$	The shortest distance from u to v.
K	A set of segment indexes of the passenger's trip.
κ	A segment index of the passenger's trip, $\kappa \in K$.
D^i_κ	The shortest distance of segment κ of r_i.
C	Capacity of shared taxi.
η	Minimum number of bus stations to be visited.
θ	A parameter for walking distance.

Table 1. Notations

2.2 Problem Formulation

An example is depicted in Figure 1 with two requests (r_1, r_2) and two bus lines (b_1, b_2). There is one passenger per request. The circles o_1 and o_2 indicate two passengers' origins, and squares d_1 and d_2 indicate the destinations. The number above an arc represents the time (minute) it takes to travel between two points. Additionally, the number in parentheses represents the distance (kilometer) between two points. The blue arc represents bus line b_1, which contains k_1, k_2, and k_3 stations. The green arc represents bus line b_2, which contains k_3, k_4, and k_5 stations. k_3 is the station that two buses pass through. The red arc represents the vehicles' trajectory route, and the black arc represents the passenger's walking route. An ordinary taxi does not offer ridesharing, whereas a shared taxi offers it. Since ridesharing potentially affects passengers' experience (e.g., the possibility of making detours), taxi charges 2.3 yuan/km (yuan is the basic unit of the official currency of China) and shared taxi charges 2 yuan/km. In addition, an appropriate discount of 0.1 is carried out for the additional detour distance of each request. The fare for each bus trip is 2 yuan per passenger. Suppose that passengers want to get from their origins to their destinations within 60 minutes. For the sake of brevity, only the distance traveled by passengers on shared taxis (or taxis) is calculated.

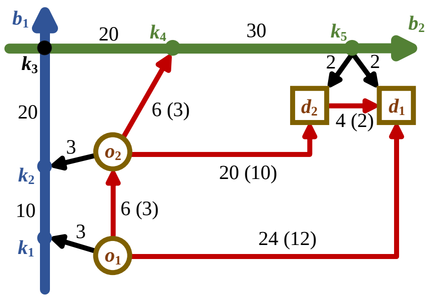

Figure 1. An example of different modes of travel for two requests from origins (circle) to destinations (square)

According to the results in Table 2, the combined travel mode should be explored to get better results. Passengers spend less money to reach their destinations within designated timeframes. The use of buses reduces the shared taxis' travel distance. The revenue per kilometer of shared taxis is larger than other modes. Consequently, this mode can be regarded as a green travel mode.

The combined travel mode is demonstrated in Figure 2. The trip of passengers is divided into three segments, denoted by K ($K = \{1, 2, 3\}$). The trip of a passenger can be categorized into five types:

1. A passenger walks to the pick-up station, then takes a bus to the drop-off station, and finally walks to its destination;
2. A passenger takes a shared taxi to the pick-up station, then takes a bus to the drop-off station, and finally walks to its destination;
3. A passenger walks to the pick-up station, then takes a bus to the drop-off station, and finally takes a shared taxi to its destination;
4. A passenger takes a shared taxi to the pick-up station, then takes a bus to the drop-off station, and finally takes a shared taxi to its destination; and
5. A passenger takes a shared taxi directly to its destination.

In particular, we consider the fifth type where the passenger has a walking distance of 0 from the origin to the shared taxi's pick-up point and from the shared taxi's drop-off point to the destination, so $D_2^i = D(o_i, d_i)$, $D_1^i = D_3^i = 0$, where D_κ^i is the shortest distance of segment κ of r_i, κ ($\kappa \in K$) is a segment index of the passenger's trip, and $D(o_i, d_i)$ is the shortest distance from o_i to d_i.

Mode	**Route**	**Cost [yuan]**	**Vehicles' Distance [km]**	**Revenue per kilometer [yuan/km]**	**Travel Time [minute]**	Feasible
Taxi	$r_1 : o_1$-d_1; $r_2 : o_2$-d_2.	$r_1 : 27.6$ $r_2 : 23.0$	22	2.30	$r_1 : 24$ $r_2 : 20$	Yes
Bus	$r_1 : o_1$-k_1 -k_3-k_5-d_1; $r_2 : o_2$-k_2 -k_3-k_5-d_2.	$r_1 : 4.0$ $r_2 : 4.0$	0	–	$r_1 : 85$ $r_2 : 75$	No
Taxi and bus	$r_1 : o_1$-k_4 -k_5-d_1; $r_2 : o_2$-k_4 -k_5-d_2.	$r_1 : 15.8$ $r_2 : 8.9$	9	2.30	$r_1 : 44$ $r_2 : 35$	Yes
Shared taxi	$r_1 : o_1$-o_2 -d_2-d_1; $r_2 : o_2$-d_2.	$r_1 : 23.7$ $r_2 : 20.0$	15	2.91	$r_1 : 30$ $r_2 : 20$	Yes
our	$r_1 : o_1$-o_2 -k_4-k_5-d_1; $r_2 : o_2$-k_4 -k_5-d_2.	r_1 :**14.0** r_2 :**8.0**	**6**	**3.00**	r_1 :**44** r_2 :**38**	**Yes**

Table 2. Comparison of different modes of travel

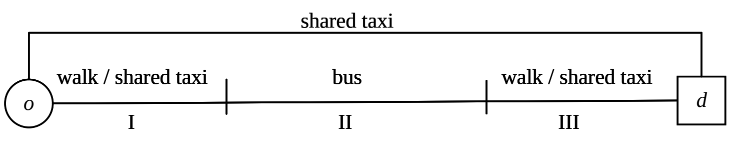

Figure 2. Segments of the combined travel mode

2.3 Mathematical Model

The goal of the problem involves three criteria defined by four decision variables. The first variable is a binary number defined as $x^j_{uv} = 1$ if s_j travels from point u to v; $x^j_{uv} = 0$, otherwise. The second variable is $y^l_i = 1$ if r_i takes b_l; $y^l_i = 0$, otherwise. The third variable is $z^\kappa_{ij} = 1$ if r_i takes s_j in segment κ; $z^\kappa_{ij} = 0$, otherwise. The fourth variable is $w^j_{uv} = 1$ if s_j carries passengers traveling from point u to v; $w^j_{uv} = 0$, otherwise.

An objective aims to minimize the total cost of all passengers, which includes the cost spent by passengers on shared taxis and buses denoted by e_i and $\tilde{e}_i$, respectively.

Suppose that the route of s_j is $\langle \ldots, u-1, u, u+1, \ldots, v-1, v, v+1 \rangle$, where u and v are the pick-up and drop-off points for r_i to take s_j on the first segment, respectively. The detour ratio of r_i on s_j is:

$$\delta_{ij} = \frac{\sum_{v=u}^{v-1} D(v, v+1) - D(u, v)}{D(u, v)}. \tag{1}$$

At the same time, the shortest distance of segment $\kappa = 1$ of r_i:

$$D_1^i = D(u, v). \tag{2}$$

For the cost spent by passengers on shared taxis, a pricing rule is designed. The charge for shared taxis per kilometer is p. Acknowledging the influence of the need for ridesharing and transfer, which harms the interests of passengers, shared taxis charge lower than ordinary taxis. An appropriate discount λ is carried out for the additional detour distance of each request. In the case of more than one passenger in a request, a fare $\overline{p}$ is charged for each additional passenger. The cost of passengers in r_i to take shared taxis is as follows:

$$e_i = \sum_{\kappa \in K} \sum_{j \in J} z_{ij}^{\kappa} \cdot \left[(p - \lambda \cdot \delta_{ij}) \cdot D_{\kappa}^i + (c_i - 1) \cdot \bar{p} \right]. \tag{3}$$

For simplicity, we charge a fare $\widetilde{p}$ for each passenger on the bus. Additionally, passengers who need to walk to access the bus are eligible for a walking discount determined by the walking discount parameter ρ. The cost of passengers in r_i to take buses is as follows:

$$\tilde{e}_i = c_i \cdot \left(\sum_{l \in L} y_i^l \cdot \tilde{p} - \sum_{\kappa \in \tilde{K}} \sum_{j \in J} \rho \cdot (1 - z_{ij}^{\kappa}) \cdot D_{\kappa}^i \right), \tag{4}$$

where $\tilde{K} = \{1, 3\}$ is a subset of K.

The first objective function is defined as follows:

$$\min f_1 = \sum_{i \in I} (e_i + \tilde{e}_i). \tag{5}$$

In addition, we also want to reduce vehicles' energy consumption and achieve green travel by minimizing the travel distance of shared taxis. The second objective function is defined as follows:

$$\min f_2 = \sum_{j \in J} \sum_{u,v \in \Omega} x_{uv}^j \cdot D(u, v), \tag{6}$$

where Ω is a set of points that shared taxis may pass by, and u and v are two points in Ω.

The total revenue of shared taxis is related to the cost of all passengers. The revenue per kilometer of shared taxis is calculated as the ratio of the total cost of passengers on shared taxis to the distance traveled by passengers on shared taxis. The third conflicting objective is set to maximize the revenue per kilometer of shared taxis.

$$\max f_3 = \frac{\sum_{i\in I} e_i}{\sum_{j\in J}\sum_{u,v\in\Omega} w_{uv}^j \cdot D(u,v)}, \tag{7}$$

s.t.

$$\sum_{j\in J} x_{uv}^j \le 1, \quad \forall u\in\Omega, \forall v\in\Omega, \tag{8}$$

$$\sum_{l\in L} y_i^l \le 1, \quad \forall i\in I, \tag{9}$$

$$\sum_{\kappa\in K}\sum_{j\in J} z_{ij}^{\kappa} \le 2, \quad \forall i\in I, \tag{10}$$

$$\sum_{j\in J} z_{ij}^{\kappa} \le 1, \quad \forall i\in I, \forall \kappa\in K, \tag{11}$$

$$\sum_{j\in J} w_{uv}^j \le 1, \quad \forall u\in\Omega, \forall v\in\Omega, \tag{12}$$

$$0 < c_i \le C, \quad \forall i\in I, \tag{13}$$

$$0 \le \varepsilon_u^j \le C, \quad \forall j\in J, \forall u\in\Omega, \tag{14}$$

$$\sum_{j\in J} z_{ij}^{\kappa} = 0, \quad \forall i\in I, \forall \kappa\in\tilde{K}, D_{\kappa}^i \le \theta, \tag{15}$$

$$\varpi_i \ge \eta, \quad \forall i\in I, \tag{16}$$

$$t_u^j - t_i \le \sigma, \quad \forall i\in I, \forall j\in J, u=o_i, \tag{17}$$

$$t_u^j - \tilde{t}_u^i \le \sigma, \quad \forall i\in I, \forall j\in J, u=\hat{k}_i, \tag{18}$$

$$\tilde{t}_u^i \le a_i, \quad \forall i\in I, u=d_i, \tag{19}$$

$$\sum_{j\in J} z_{ij}^{\kappa} \cdot \sum_{l\in L} y_i^l = 0, \quad \forall i\in I, \kappa=2, \tag{20}$$

$$x_{uv}^j \in \{0,1\}, \quad \forall j\in J, \forall u\in\Omega, \forall v\in\Omega, \tag{21}$$

$$y_i^l \in \{0,1\}, \quad \forall l\in L, \forall i\in I, \tag{22}$$

$$z_{ij}^{\kappa} \in \{0,1\}, \quad \forall \kappa\in K, \forall i\in I, \forall j\in J, \tag{23}$$

$$w_{uv}^j \in \{0,1\}, \quad \forall u\in\Omega, \forall v\in\Omega, \forall j\in J. \tag{24}$$

Constraints (8) and (12) concern at most one shared taxi pass between any two points. Constraint (9) restricts that each passenger takes no more than one bus. Considering that too many transfers affect the passenger experience, transfers between two buses are not allowed. Constraint (10) determines that each passenger takes at most two shared taxis to reach its destination. According to Constraint (11), each passenger takes at most one shared taxi in a particular segment. Constraint (13) ensures that the number of passengers in a request is less than or equal to the shared taxi's capacity C. Constraint (14) requires that the number of passengers in s_j at u, denoted by ε_u^j, does not exceed the shared taxi's capacity. Constraint (15) guarantees that if the distance between the passenger's origin and pick-up station (or between the passenger's drop-off station and destination) is less than θ, the passenger needs to walk. Constraint (16) ensures the number of bus stations visited by a passenger, denoted by ϖ_i, must not be less than the minimum number of bus stations to be visited, denoted by η. Otherwise, it is meaningless to take the bus. Constraints (17) and (18) define that passengers do not wait more than σ minutes for a shared taxi where t_u^j is the time s_j arrives at u and $\tilde{t}_u^i$ is the time r_i arrives at u. They guarantee that passengers with a closer origin or similar dispatch time are assigned to the same shared taxi. Constraint (19) guarantees that all passengers need to arrive at their destination within designated timeframes. Constraint (20) ensures that if a passenger takes a shared taxi for the second segment, the bus is not taken, and vice versa. Constraints (21), (22), (23) and (24) define the range of decision variables.

These constraints play a vital role in ensuring that the problem follows specific rules and requirements during the optimization process, resulting in the solution of the problem conforming to the intended requirements.

3 METHODOLOGY

We first divide the area where the requests are distributed into several small regions. A set of bus lines are selected from this area, which can connect to each region. Those requests originating from the same region are categorized into the same group, and the optimization algorithm is uniformly called for planning. Then, each group of requests is calculated in batches. Additionally, we set up some parking lots in each region for non-working shared taxis to park.

3.1 Encoding and Decoding

1) **Encoding:** For instance, there are five requests: r_1–r_5; four shared taxis s_1–s_4; two bus lines b_1 and b_2, and each bus line has ten stations. An integer encoding is adopted to represent each individual. Let 1–4 denote the four shared taxis, 5–14 denote the ten stations of b_1, and 15–24 denote the ten stations of b_2. Additionally, we employ 0 to denote that passengers do not take shared taxis or buses. To ensure a high diversity of solutions, we randomly initialize a quadruple for each request and aggregate these quadruples into a matrix. Each

quadruple represents a request's travel mode. The row of this matrix represents the request's index, and the number of rows represents the number of requests. Figure 3 represents two individuals (φ_1 and φ_2).

2	6	13	0
1	7	14	0
0	18	24	3
1	0	0	0
2	0	0	0

2	16	23	3
2	18	24	3
0	8	14	0
1	6	14	4
1	0	0	0

Figure 3. Two initialized individuals: φ_1 (left) and φ_2 (right)

2) Decoding: The matrix depicts each request's trip planning and each shared taxi's travel route. The individual φ_1 is taken as an example. It depicts five requests' trip planning:

Row 1: The passengers of r_1 take s_2 to station 6, then take b_1 to station 13, and finally walk to their destinations.

Row 2: The passengers of r_2 take s_1 to station 7, then take b_1 to station 14, and finally walk to their destinations.

Row 3: The passengers of r_3 walk to station 18, then take b_2 to station 24, and finally take s_3 to their destinations.

Row 4: The passengers of r_4 directly take s_1 to their destinations.

Row 5: The passengers of r_5 directly take s_2 to their destinations.

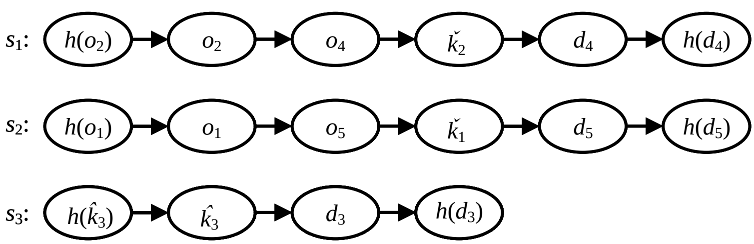

Figure 4. The route of each shared taxi in individual φ_1

Each shared taxi's travel route in an individual is obtained as follows. Based on the individual, the possible points that each shared taxi can pass through are identified. Those points are classified according to five categories: parking lots, origins, destinations, pick-up stations, and drop-off stations. Firstly, the nearest parking lot is selected to schedule an idle shared taxi departure. Subsequently, on the basis of looking at the next closest point, the shared taxi travels origins, pick-up stations, drop-off stations, and destinations. Ultimately, the shared taxi returns to a parking lot closest to its last point. Each shared taxi's route is obtained by decoding φ_1, which is shown in Figure 4. We denote by $h(u)$ $(u \in \Omega)$ the parking lot closest to point u. Based on each shared taxi's route, passengers' transfer time and arrival time at their destinations are calculated. Therefore, the idle or loaded situation of each shared taxi is obtained.

3.2 NLSGA Algorithm

Genetic algorithm is a search algorithm produced by the theory of evolution and genetic mechanism [33]. It keeps individuals in the population unlike each other by evolution. Those individuals who adapt to their environment are more likely to survive, reproduce, and pass on their traits to the next generation. In this section, we present the nondominated linear sorting genetic algorithm (NLSGA) to address the multi-modal route planning problem, which can achieve the matching of shared taxis (or buses) and passengers and the optimization of shared taxis' routes. NLSGA introduces three significant improvements to tackle this problem. Firstly, in terms of population initialization, NLSGA employs targeted methods, such as selecting appropriate buses and pick-up and drop-off stations for each passenger, whereas other algorithms rely on random initialization. Secondly, NLSGA incorporates two crossover operators during population evolution, namely global crossover between different individuals and local self-crossover within the same individual. The former helps optimize the overall route planning of shared taxis, while the latter focuses on the local optimization of passengers' trip planning. In contrast, other algorithms use random crossover, which complicates the search for optimal solutions and increases the likelihood of getting trapped in local optima. Finally, with regard to the population selection, other algorithms conduct a global search throughout the entire iteration cycle. NLSGA introduces a linear ranking selection strategy. In the early iterations, it emphasizes global search to maintain population diversity. As the number of iterations increases, the selection concentrates on high-performing individuals. This adaptive adjustment enables NLSGA to conduct more detailed local searches around the current optimal solution, accelerating convergence and enhancing the quality of the solution. Figure 5 depicts the proposed approach. Four essential procedures are as follows:

1. Initialization of the population;
2. Population evolution;
3. Population repairing; and
4. Population selection.

Algorithm 1 demonstrates this pseudocode. Next, the details of the algorithm are described.

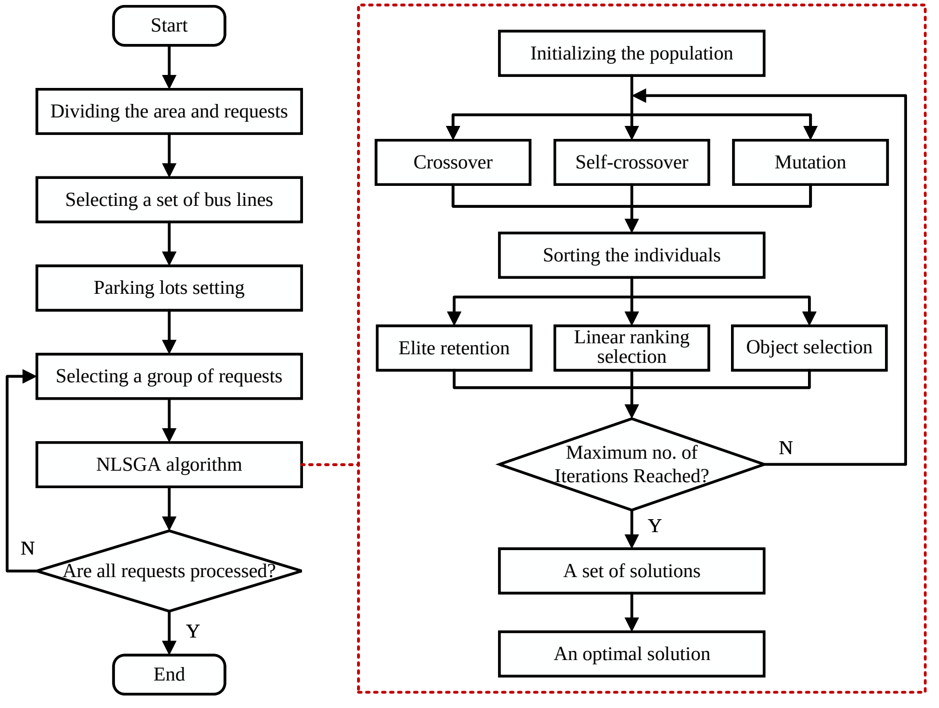

Figure 5. Flowchart of the proposed approach

1) **Initialization of the population:** We take r_i as an instance. First, an appropriate bus needs to be chosen for r_i. It is categorized into the following three steps:

Circle drawing: We connect the origin and destination of r_i and draw a straight line. A circle is drawn with the midpoint of the line as the center and half of the line's distance as the radius.

Bus selection: It should check which buses have more than η bus stations located within this circle. Those buses that meet this condition are added to a bus list. If the bus list is not empty, we randomly choose a bus for r_i; otherwise, there is no eligible bus for r_i, and we directly arrange a shared taxi to take the passengers of r_i to the destination.

Pick-up and drop-off station selection: If r_i is arranged a bus, a suitable pair of pick-up and drop-off stations needs to be selected. We use the nearest strategy: For the pick-up station, the three-nearest stations to the origin are considered. If there exists a station whose distance from the origin does not exceed the distance parameter θ, we choose this station as the pick-up station, and passengers walk from the origin to the pick-up station. How-

Algorithm 1 NLSGA.

Input: Population size $\check{N}$, maximum iteration count $\hat{G}$, two linear selection parameters τ^- and τ^+, a selection parameter $\tilde{n}$, and the probabilities of the three operations.
Output: An optimal solution.
Initialize a population P_1;
Calculate the objective function values for each individual in P_1;
for $\hat{g} = 1, 2, \ldots, \hat{G}$ **do**
    Let $P_{\hat{g}}$ execute the three operations to obtain $\tilde{P}_{\hat{g}}$;
    Repair the individuals in $\tilde{P}_{\hat{g}}$ that do not satisfy the constraints;
    Calculate the objective function values for the individuals in $\tilde{P}_{\hat{g}}$;
    Merge the new population as $P = P_{\hat{g}} \cup \tilde{P}_{\hat{g}}$;
    Perform a non-dominated sorting and crowding distance calculation on the individuals in P according to f_1 and f_2;
    Choose the top $\acute{n}$ individuals and denote them as $\bar{P}_{\acute{n}}$;
    Select the top $\tilde{n}$ individuals on f_3 and denote them as $\hat{P}_{\tilde{n}}$;
    Alter the values of τ^- and τ^+;
    Rank the remaining $N(N = 2\check{N} - \acute{n} - \tilde{n})$ individuals according to f_3 and assign the selection probabilities;
    Linearly select $\check{N} - \acute{n} - \tilde{n}$ individuals and denote them as $\check{P}_{\check{N}-\acute{n}-\tilde{n}}$;
    Merge the new population as $P_{\hat{g}+1} = \bar{P}_{\acute{n}} \cup \hat{P}_{\tilde{n}} \cup \check{P}_{\check{N}-\acute{n}-\tilde{n}}$;
    $\hat{g} = \hat{g} + 1$;
end for
Obtain a set of solutions;
Select an optimal solution according to the priority order: f_3, f_1 and f_2.

ever, if no such station is found, we randomly choose a station from the three-nearest stations. Subsequently, an appropriate shared taxi is chosen to deliver r_i from the origin to the pick-up station. The same approach is applied to the drop-off station's selection.

In addition, there is the selection of shared taxis. It includes two types: one from an origin to a pick-up station (or directly from an origin to a destination), and the other from a drop-off station to a destination. Some idle shared taxis are filtered from different parking lots. For a request to board at its origin, a shared taxi is chosen from the nearest parking lot in the region where the origin is located. Likewise, for the request to board at the drop-off station, a shared taxi is chosen from the nearest parking lot in the region where the drop-off station is located. They all dispatch on the basis of satisfying the capacity constraint of the shared taxi.

2) Population evolution: The important drivers of evolution are crossover and mutation [34]. Crossover helps maintain the diversity of the population and fuses better features together over time. In this section, two new crossover operators are designed, including globally optimized crossover among different individuals and locally optimized self-crossover within the same individual [35]. The crossover operation is performed with probability. All crossover operations are performed on the basis of satisfying the capacity constraint of the shared taxi. The first crossover operation helps to enhance the algorithm's global optimization ability. A subset of requests is randomly selected. On the basis of satisfying constraints, these requests' quadruples are swapped between two individuals. In Figure 6, r_4 and r_5 in two individuals are swapped. In addition, Figures 7 a) and 7 b) illustrate self-crossover, which prevents the algorithm from falling into the local optimum. Two requests are selected randomly, which consists of two cases: a) Passengers in two requests are served by a shared taxi delivery to the same bus, and their pick-up stations differ by no more than three stations. We arrange for these passengers to take the same shared taxi to the same pick-up station. The same situation also applies to the drop-off station. It is described as a non-nearest strategy. As shown in Figure 7 a), r_1 and r_2 are on the same bus, and their pick-up stations differ by one station. The shared taxi of r_2 is replaced with s_2, and the pick-up station of r_2 is replaced with 6. b) If passengers in two requests are sent to their destinations directly through shared taxis, we replace their shared taxis with the same ones. In Figure 7 b), r_4 and r_5 directly take shared taxis to reach their destinations. The shared taxi of r_5 is replaced with s_1.

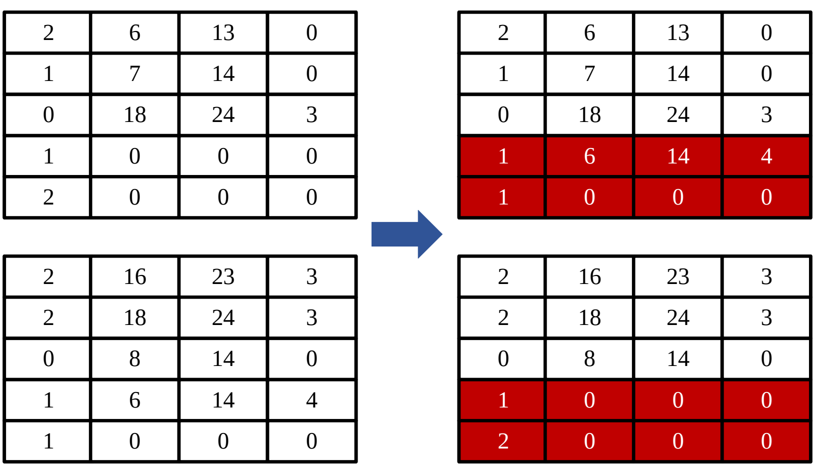

Figure 6. The first crossover operation

Moreover, mutation plays an important role in evolution by introducing accidental changes. On the basis of satisfying the capacity constraint of the shared taxi, we randomly alter an element's value in the request's quadruple, as shown in Figure 8. By introducing randomness, it increases the possibility of obtaining better individuals.

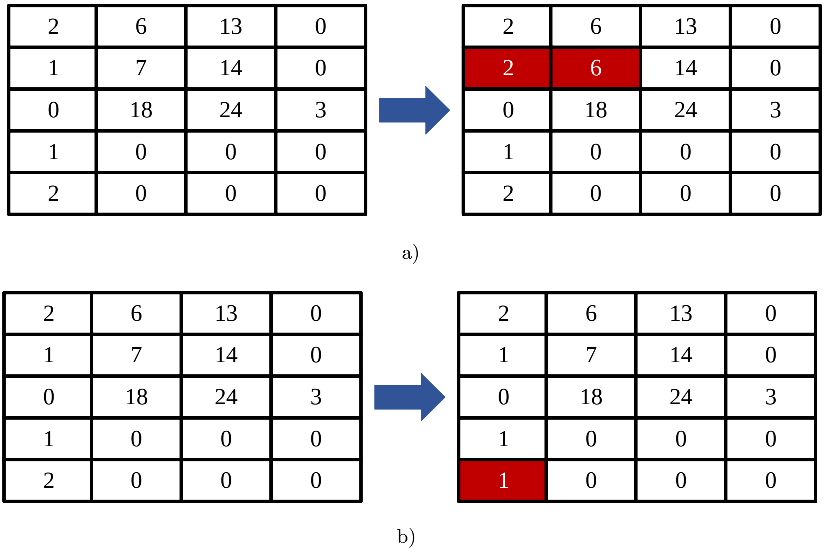

Figure 7. Two self-crossover operations

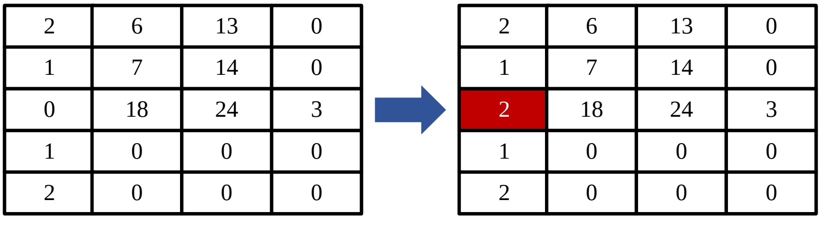

Figure 8. Mutation operation

3) Population repairing: The process of population repairing is described in Algorithm 2. It repairs individuals in the population slightly and helps individuals who do not reach their destination within designated timeframes to arrive on time.

4) Population selection: Consider that f_3 is related to f_1 and f_2. A selection method combining an elite retention strategy and the linear ranking selection strategy [36] is adopted. Suppose that $\check{N}$ individuals are chosen from the parent population ($2\check{N}$ individuals).

Step 1: The elite retention strategy is adopted. According to the non-dominated sorting and crowding distance calculation methods, the top $\acute{n}$ individuals are selected.

Algorithm 2 Repairing.

Input: Population size $\check{N}$, request size n, a parameter η, population, dataset (request dataset, distance dataset, bus line dataset).
Output: The repaired population.
for $\check{n} = 1, 2, \ldots, \check{N}$ **do**
  for $i = 1, 2, \ldots, n$ **do**
    if r_i needs to take the bus **then**
      Count the number of bus stations taken by r_i and denote it as ϖ_i;
      if $\varpi_i \leq \eta$ **then**
        Schedule a shared taxi for r_i and take the passengers of r_i directly from the origin to the destination;
      end if
    end if
    Calculate the travel time of r_i;
    if r_i does not arrive on time **then**
      if r_i needs to take the bus **then**
        Schedule an empty shared taxi for its first segment, and recalculate the travel time of r_i ;
        if r_i still does not arrive on time **then**
          Schedule an empty shared taxi for its third segment, and recalculate the travel time of r_i;
          if r_i still does not arrive on time **then**
            Reserve the shared taxi of the first segment and take the passengers of r_i directly from the origin to the destination;;
          end if
        end if
      else
        Schedule an empty shared taxi for r_i and take the passengers of r_i directly from the origin to the destination;
      end if
    end if
  end for
end for

Step 2: The selection process retains the top $\tilde{n}$ individuals with the highest revenue per kilometer of shared taxis. To maintain population diversity, the parameter $\tilde{n}$ is set to ten.

Step 3: $\check{N} - \acute{n} - \tilde{n}$ individuals are selected from the remaining parent population (which has N ($N = 2\check{N} - \acute{n} - \tilde{n}$) individuals) by the linear ranking selection strategy: a) The revenue per kilometer of shared taxis is utilized to rank the remaining N individuals from small to large, and the selection probabilities are assigned. b) We sequentially select the $\check{N} - \acute{n} - \tilde{n}$ individuals according to their assigned probabilities. Assume that individual φ_1 is the highestranked individual, and individual φ_N is the lowest. The probability that an individual φ_μ is selected is defined as:

$$\epsilon_\mu = \frac{\tau^- + (\tau^+ - \tau^-)^{\frac{\mu-1}{N-1}}}{N}, \tag{25}$$

where $\mu = 1, 2, 3, \ldots, N$, τ^- and τ^+ are constants specified.

$$\tau^+ + \tau^- = 2, \tag{26}$$

$$0 \leq \tau^- \leq 1, \tag{27}$$

$$1 \leq \tau^+ \leq 2. \tag{28}$$

The selection pressure refers to the degree to which individuals with superior performance are favored and retained in an evolutionary algorithm. Notice that when $\tau^- = 0$ and $\tau^+ = 2$, the population's selection pressure is the maximum. The probability of being selected gradually increases as the rank becomes lower. In the case of $\tau^- = \tau^+ = 1$, the population's selection pressure is minimal. Each individual in the population has the same selection probability, which is random selection. τ^- and τ^+ gradually alter as the population iterates. This method can ensure the high convergence speed of the algorithm. In the algorithm's early iteration, we set $\tau^- = \tau^+ = 1$ to ensure the algorithm's searchability. It can expand the search space and avoid falling into local optimum. In the iteration process, the value of τ^- is gradually reduced, and the value of τ^+ is gradually increased. It can improve the selection pressure of the population and ensure the convergence of the algorithm; thus, the algorithm can better approach the optimal value. Ultimately, $\tau^- = 0$ and $\tau^+ = 2$. The population maintains the maximum pressure selection, which helps the population to converge.

4 EXPERIMENTAL RESULTS

In this section, the experimental settings are first introduced. Then, the convergences, comparisons with several existing algorithms, and other results to evaluate the performance of the NLSGA are presented.

4.1 Experimental Settings

Experimental settings include the division of regions, shared taxi settings, bus settings, request data and parameter settings.

1) **Division of regions:** In this work, an area of 18 km × 20 km in Beijing is used. This area is categorized into nine regions, as depicted in Figure 9. The dashed line represents our partition scenario. Additionally, the requests in this area are categorized into nine groups. The requests originating from the same region are divided into the same group.

2) Shared taxi settings: All shared taxis take the shortest route between any two given points, and their speed is set to 30 km/h. Shared taxis are initially distributed in different parking lots.

3) Bus settings: Five bus lines are chosen for research purposes, as depicted in Figure 9. Each bus line consists of approximately 30 stations and spans about 20 km. Each bus line operates on both the forward and reverse routes, covering two directions for the bus line. The station and scheduling information are obtained based on historical real data. It is available at the link: `https://bjbus.jinzihao.me`. In reality, if the bus is delayed, the driver will take certain measures to reduce the delay as much as possible. For example, it can adjust the speed of the bus. So we assume that buses arrive on time according to their schedule.

4) Request data: The request data comes from Beijing taxi trajectory data [37, 38]. In the following experiments, we first test the case of 60 requests on a given day with a time window of T (10:06-10:10). These requests' origins are within region 2. The driving distance is calculated using Baidu Maps' API based on their latitude and longitude coordinates.

5) Parameter settings: The price of a shared taxi is set to 2 yuan/km. If the passenger number of a request exceeds one, an additional seat fee of 2 yuan is charged. The bus charges 2 yuan per passenger. The passenger's walking distance parameter value is set to 1 km. It is the best result obtained when comparing with different walking distance parameters. Detailed comparison is shown in the next section: Different walking distance parameters.

The experimental results are obtained using a computer system configured with the following software and hardware: MATLAB 2022a, Windows 10 operating system, GPU with 2 × CPU-4216, 4 × 32 GB DDR4 2666 ER, 2 × 2 TB SATA hard disks. The results of the NLSGA are compared with various well-known optimization algorithms, including NSGA-II, MOPSO [39], and MOGWO [40].

The time complexity of NLSGA is mainly affected by the population size ($\hat{N}$), the number of objective functions ($\hat{M}$), and the number of iterations ($\hat{G}$). Since these operations are performed on individuals, the time complexity of both population initialization and evolution is $O(\hat{N})$. In the population selection phase, the time complexity of fast non-dominated sorting is $O(\hat{N}log\hat{N})$, the time complexity of crowding distance calculation is $O(\hat{N})$, and the time complexity of linear sorting selection is also $O(\hat{N})$. In addition, NLSGA evaluates the fitness of each individual, with an approximate time complexity of $O(\hat{N} \times \hat{M})$. Therefore, the overall time complexity of NLSGA can be expressed as $O(\hat{G} \times \hat{N}log\hat{N})$. Similarly, the time complexity of NSGA-II and MOPSO is also $O(\hat{G} \times \hat{N}log\hat{N})$, while the time complexity of MOGWO is $O(\hat{G} \times \hat{N} \times \hat{M})$. The population size and the number of iterations are set to be the same for these optimization algorithms. Although NLSGA does not have a significant advantage in terms of time complexity, it outperforms other algorithms in the following experimental results.

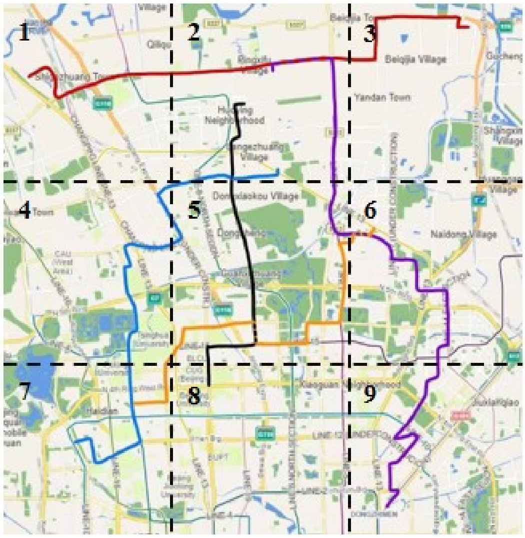

Figure 9. Experimental area and five bus lines

4.2 Results

Comparison before and after optimization: As shown in Figure 10, the left is the objective value before optimization (the un-optimized initial population), and the right is the objective value after optimization. It shows that after optimization, the values of f_1 and f_2 are reduced, and the value of f_3 is increased.

In addition, we compare different optimization algorithms using the random initialization method, and NLSGA (with our initialization method) to execute five repeated experiments. Each experiment measures the average values of the three objective functions in the un-optimized population and the optimized population, respectively. The experimental results are shown in Table 3. It can be seen that in the case of the random initialization method, NLSGA (with the random initialization method) outperforms other algorithms in terms of the total cost of passengers and the total travel distance of shared taxis. At the same time, NLSGA (with our initialization method) seems to perform better than other algorithms in the overall performance of the three objective functions, achieving lower total cost of passengers and shorter travel distance of shared taxis while also maintaining significant revenue per kilometer of shared taxis. Moreover, in the five repeated experiments performed by NLSGA (with our initialization method), there is minimal difference in the performance of the three objective functions between the un-optimized population and the optimized population.

Convergences of the two objective functions: Figure 11 illustrates the convergences of the three objective functions. It can be seen that the three objective

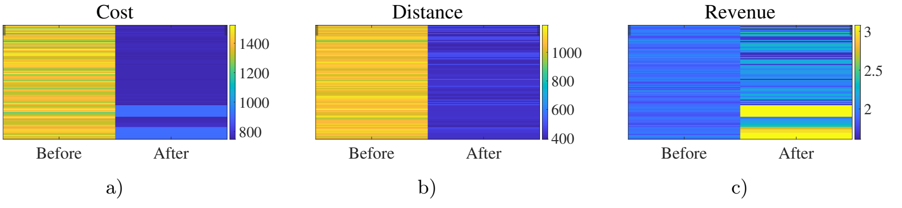

Figure 10. Population before and after optimization

functions reach the optimal value within 400 runs. Finally, the NLSGA gradually tends to converge, i.e., the 500th iteration.

Comparison of different optimization algorithms: The NLSGA is compared with existing optimization algorithms on three objective functions. Table 4 depicts

The un-optimized population

		Cost [yuan]	Distance [km]	Revenue per Kilometer [yuan/km]
Random initialization method	NLSGA	1 695.957	1 316.667	1.936135
	NSGA-II	1 696.641	1 315.458	1.930132
	MOPSO	1 669.103	1 293.047	1.914252
	MOGWO	1 692.342	1 317.722	1.927431
NLSGA (with our initialization method)	Round 1	1 358.941	1 050.378	1.932567
	Round 2	1 361.935	1 051.458	1.916954
	Round 3	1 365.407	1 058.005	1.920127
	Round 4	1 363.644	1 054.312	1.917231
	Round 5	1 363.784	1 058.849	1.913773

The optimized population

		Cost [yuan]	Distance [km]	Revenue per Kilometer [yuan/km]
Random initialization method	NLSGA	931.142	539.958	2.249599
	NSGA-II	986.587	589.396	2.271405
	MOPSO	1 140.883	701.664	2.526194
	MOGWO	1 102.566	712.752	2.502885
NLSGA (with our initialization method)	Round 1	806.851	472.976	2.261133
	Round 2	816.836	471.116	2.240904
	Round 3	805.907	446.961	2.310207
	Round 4	804.296	467.857	2.252454
	Round 5	796.592	442.343	2.313265

Table 3. The comparison of initialization

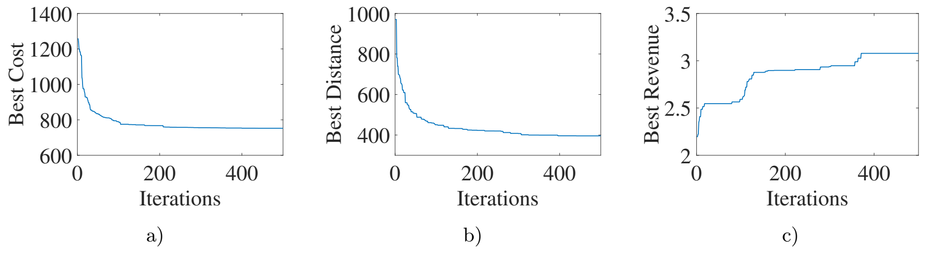

Figure 11. Convergences of the three objective functions

the optimal solution obtained by different algorithms. Table 5 depicts the average values of the optimized population. It can be seen that compared with other optimization algorithms, NLSGA achieves better results on the three objective functions.

Algorithm	**Cost [yuan]**	**Distance [km]**	**Revenue per Kilometer [yuan/km]**
NLSGA	**915.4015**	**429.848**	**3.0781**
NSGA-II	1008.565	602.004	2.627048
MOPSO	1145.446	683.817	2.640421
MOGWO	1271.291	771.075	2.548671

Table 4. The optimal solution of different optimization algorithms

Algorithm	**Average Cost [yuan]**	**Average Distance [km]**	**Average Revenue per Kilometer (yuan/km)**
NLSGA	**795.4120**	**436.0583**	**2.245892**
NSGA-II	986.5872	589.3966	2.271405
MOPSO	1140.883	701.6643	2.526194
MOGWO	1341.566	812.752	2.562885

Table 5. The average values of the optimized population

Different walking distance parameters: We take seven different walking distance parameters θ. Under different values, the experimental results are based on 20 runs, with each run consisting of 500 iterations. In each run, the average values of the three objective functions for an optimized population are recorded. Figure 12 illustrates the three objective functions' variations for different θ. The blue dots represent the average value obtained by averaging these 20 values. In the initial stage, with the increase in walking distance, the total cost of passengers and the total travel distance of shared taxis are gradually reduced, and the revenue per kilometer of shared taxis is increasing. However, as the walking distance increases, the passenger's travel time also increases. Since there is a limit to the latest arrival time, passengers need to take shared taxis to reduce their travel time once the limit is exceeded. It is worth noting that the three objectives' values gradually level off when the θ exceeds 1. To consider passengers' travel experience, the case $\theta = 1$ in our experiments is adopted.

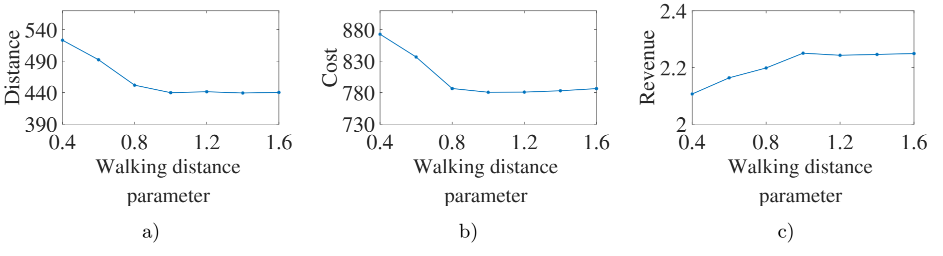

Figure 12. Comparison of different walking distance parameters

Comparison of different strategies for selecting the pick-up and drop-off stations: Figure 13 shows the comparison of the two cases: one is the nearest strategy and the other is the non-nearest strategy. The black and blue boxes represent the nearest strategy and the non-nearest strategy, respectively. The experimental results are based on 20 runs, each consisting of 500 iterations. For each run, the average values of the three objectives for the optimized population are recorded. The figure displays the distribution of the three objectives' average values under two cases. It can be seen that the total cost of all passengers and travel distance of shared taxis is shortened, and the revenue per kilometer of shared taxis is improved after executing our designed non-nearest strategy.

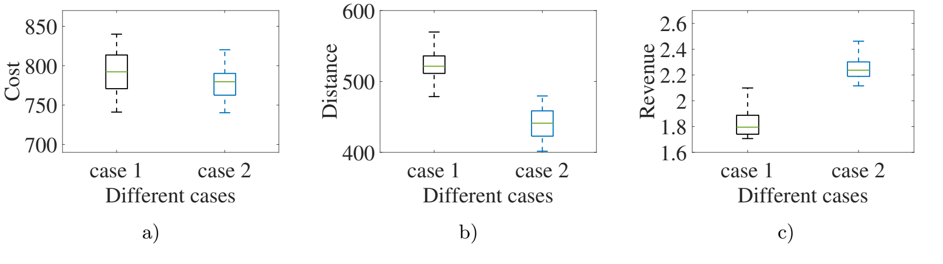

Figure 13. Comparison of different strategies for selecting the pick-up and drop-off stations

Large-scale experiment: We conduct an experiment involving requests' origins from region 2 over a one-hour period (specifically from 10:01 to 11:00), resulting in a total of 1 664 requests. Figure 14 depicts the distribution of the destinations for these requests across nine regions. To manage the schedule effectively, we divided it into time periods of five minutes each, and Table 6 provides a comprehensive summary of the outcomes for these 12 time periods. The distance calculation includes multiplying the shortest distance by a random coefficient ranging from 1.2 to 1.6. Other parameter settings are consistent with the above experiments. In addition, the optimal results of four travel modes are compared, including taxi (without ridesharing), combining taxi and bus, shared taxi (ridesharing), and combined travel mode. Table 7 shows the optimal results of the four modes. Compared with only using taxis, the designed combined travel mode significantly reduces the travel cost by 51.30% in terms of passengers and improves the revenue per kilometer of shared taxis by 49.08% in terms of shared taxi operators. At the same time, the travel distance of shared taxis is greatly shortened and the travel distance is reduced by 56.52%, which further promotes green travel. The results illustrate the superiority of this combined travel mode.

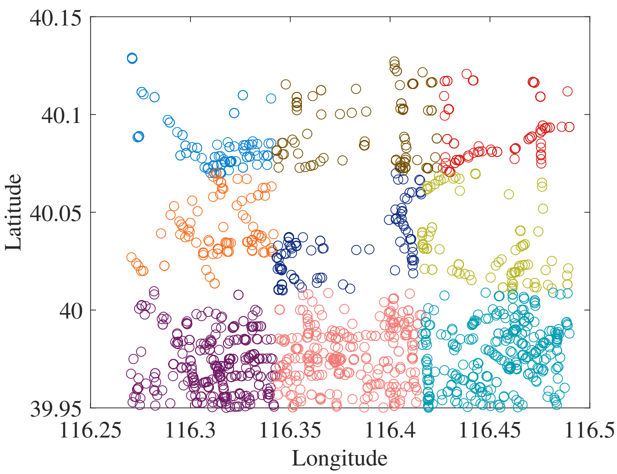

Figure 14. The distribution of requests' destinations in the nine regions

5 CONCLUSION

In order to promote green travel, this work proposes a combined travel mode combining shared taxis and buses. A multi-objective optimization model is established to minimize the total cost of all passengers and the total travel distance of all shared taxis and maximize the revenue per kilometer of shared taxis. The interests of passengers and shared taxi operators are considered simultaneously. At the

Time	**Request Number**	**Cost [yuan]**	**Distance [km]**	**Revenue per Kilometer [yuan/km]**
10:01–10:05	120	1 729.2364	720.0119	3.205944
10:06–10:10	135	2 135.8412	853.0081	3.282733
10:11–10:15	174	3 550.3889	1 281.5190	3.781144
10:16–10:20	138	2 727.7233	1 036.7790	3.515942
10:21–10:25	127	2 537.6844	959.8821	3.613317
10:26–10:30	140	2 440.0793	943.2036	3.410153
10:31–10:35	125	2 029.9665	823.0468	3.286832
10:36–10:40	150	2 718.6771	1 096.3990	3.280213
10:41–10:45	152	2 634.4403	1 086.7870	3.217046
10:46–10:50	153	2 740.0647	1 020.9410	3.661855
10:51–10:55	134	2 493.3703	954.6136	3.442129
10:56–11:00	116	2 166.1400	831.2711	3.448720
total	1 664	29 903.6120	11 607.4622	3.428836 (average)

Table 6. Numerical results for different time periods

Mode	**Cost [yuan]**	**Distance [km]**	**Revenue per Kilometer [yuan/km]**
Taxi	61 402.9481	26 696.9334	2.30000
Taxi and bus	33 885.1800	25 011.9251	2.30000
Shared taxi	54 351.8956	17 777.4783	3.398560
Combined travel (our)	**29 903.6120**	**11 607.4622**	**3.428836 (average)**

Table 7. Comparison of numerical results for different travel modes

same time, a new pricing rule is designed to support this combined travel mode. In addition, a new NLSGA is proposed. The simulations based on Beijing taxi data sets verify the effectiveness of the proposed algorithm. This combined travel mode effectively merges the flexibility of shared taxis with the stability of public transportation, reducing the travel cost of passengers while enhancing the accessibility and adaptability of public transportation. Additionally, it expands existing travel options – such as Amap's ability to offer simple recommendations for combined taxis and buses journeys – by providing a seamless connection between shared taxis and buses, along with a ridesharing service that further optimizes the travel experience. From an economic perspective, this mode not only lowers the travel cost of passengers but also boosts the profitability of operators by increasing the revenue per kilometer of shared taxis, thereby promoting the development of the local economy. Environmentally, the combined travel mode reduces reliance on private cars, alleviates traffic congestion, and decreases carbon emissions, leading to improved air quality. Overall, this comprehensive approach strongly supports sustainable urban transportation.

There are several restrictions and assumptions to consider in the combined travel mode of shared taxis and buses. Firstly, planning often depends on static demand predictions, which may not accurately reflect dynamic changes in traffic flow and passenger demand. This can result in uneven resource allocation and difficulties in addressing peak times or emergencies. Additionally, as cities expand, the scalability of current modes faces challenges, highlighting the need for further research to maintain the system efficiency and flexibility in complex traffic environments. Future directions include developing a real-time adaptive scheduling system that utilizes big data and machine learning to dynamically adjust shared taxis' planning based on real-time traffic data. This approach aims to improve service responsiveness and passenger satisfaction while also exploring the integration of multi-modal travel with intelligent transportation infrastructure to tackle complex urban traffic issues.

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